Advantages and Disadvantages of Spectral Method

Advantages

- Derivatives computed exactly
- Infinite convergence rate in space (in term of the order of accuracy)
- Can pick basis functions that are well-suited for the particular problem, e.g., spherical harmonics for flow on a sphere (the negative side is that one does not have complete freedom to choose the basis function – often subject to certain limitation such as the periodicity condition)
- Can obtain power spectra directly
- Can control aliasing, therefore NL instability easily
- Can apply spatial filters of very high order easily
- Often more accurate than FD method with the same number of degrees of freedom (grid points versus spectral components)
- Conserves energy naturally (see Haltiner and Williams section 6.3)

Disadvantages

- More complicated to implement

- Can't represent physical processes in spectral space
- Hard to parallelize on distributed memory computers
- Basis function global, not well suited for handling localized features and/or sharp gradients (remember the Gibbs phenomenon). FEM and those based on local basis functions usually do better
- Most global models since 70s are spectral models.
- Spectral methods are expensive at high resolutions. The operation count for FFT is proportional to $N \ln(N)$ instead of N as for grid point method – one of the reasons (the dominance of distributed-memory parallel computers is another) why grid point method is coming back in fashion for global models.
- Few regional model uses spectral method because of the lack of natural periodic boundary condition.