TABLE 10-1

Values of the Amplitude λ^1 and Phase Error $\tilde{c}_\phi/U$ per Time Step as a Function of Wavelength for Different Computational Approximations to the Advection Equation $\partial\phi/\partial t = -U\,\partial\phi/\partial x$

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Scheme		Wavelength	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
I. Forward-in-time linear interpolation upstream	λ	$2\Delta x$	0.998	0.980	0.800	0.600	0.400	0.200	0.000	0.200	0.400	0.600	0.800	1.000	$\lvert\lambda\rvert > 1$
		$4\Delta x$	0.999	0.990	0.906	0.825	0.762	0.721	0.707	0.721	0.762	0.825	0.906	1.000	
		$10\Delta x$	1.000	0.998	0.983	0.969	0.959	0.953	0.951	0.953	0.959	0.969	0.983	1.000	
		$20\Delta x$	1.000	1.000	0.996	0.992	0.990	0.988	0.988	0.988	0.990	0.992	0.996	1.000	
	$\tilde{c}_\phi/U$	$2\Delta x$	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.667	1.429	1.250	1.111	1.000	
		$4\Delta x$	0.637	0.643	0.704	0.780	0.859	0.936	1.000	1.043	1.060	1.055	1.033	1.000	
		$10\Delta x$	0.936	0.937	0.953	0.968	0.981	0.992	1.000	1.005	1.008	1.008	1.005	1.000	
		$20\Delta x$	0.984	0.984	0.988	0.992	0.995	0.998	1.000	1.001	1.002	1.002	1.001	1.000	
II. Centered-in-time centered-in-space (leapfrog)	λ	$2\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	$\lvert\lambda\rvert > 1$
		$4\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
		$10\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
		$20\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
	$\tilde{c}_\phi/U_{\text{physical mode}}$	$2\Delta x$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
		$4\Delta x$	0.637	0.637	0.638	0.641	0.647	0.655	0.667	0.683	0.705	0.738	0.792	1.000	
		$10\Delta x$	0.935	0.935	0.936	0.938	0.940	0.944	0.950	0.956	0.964	0.974	0.986	1.000	
		$20\Delta x$	0.984	0.984	0.984	0.984	0.985	0.986	0.988	0.989	0.991	0.994	0.997	1.000	

III. Forward-in-time upstream spline interpolation	λ	$2\Delta x$	1.000	0.999	0.944	0.792	0.568	0.296	0.000	0.296	0.568	0.792	0.944	1.000	1.000
		$4\Delta x$	1.000	1.000	0.997	0.989	0.981	0.975	0.972	0.975	0.981	0.989	0.997	1.000	0.888
		$10\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996
		$20\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\bar{c}_\phi/U$	$2\Delta x$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		$4\Delta x$	0.955	0.955	0.958	0.967	0.979	0.980	1.000	1.007	1.009	1.008	1.005	1.000	1.042
		$10\Delta x$	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.001
		$20\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
IV. Adam-Bashford centered-in-space[2]	λ	$2\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	Computational mode is unstable for at least one of the wavelengths			
		$4\Delta x$	1.000	1.000	1.000	0.999	0.997	0.991	0.977	0.950	0.893				
		$10\Delta x$	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.994	0.990				
		$20\Delta x$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999				
	$\bar{c}_\phi/U_{\text{physical mode}}$	$2\Delta x$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	Computational mode is unstable for at least one of the wavelengths			
		$4\Delta x$	0.637	0.637	0.637	0.637	0.638	0.642	0.642	0.661	0.674				
		$10\Delta x$	0.935	0.935	0.935	0.936	0.936	0.937	0.938	0.941	0.945				
		$20\Delta x$	0.984	0.984	0.984	0.984	0.984	0.984	0.984	0.984	0.984				

[1] It should be noted that in an approximate scheme which is damping (i.e., $|\lambda| < 1$), reducing Δt for the same Δx does not necessarily result in less total damping after a period of time. This results because the solution technique is used more frequently during that time because of the smaller Δt. Therefore, for improved accuracy and computational efficiency, as large a Δt as permitted by the linear stability criteria, should be chosen when an approximation scheme has computational damping (computed by Charlie Martin and Jeff McQueen).

[2] The values for the Adams–Bashford scheme were computed by Alex Costa and Sue van den Heever.