Barotropic instability (horizontal shear instability)

Stage 1 of genesis for most ~~Atlantic~~ Atlantic tropical cyclones is associated with easterly waves. But where do easterly waves come from? They form over Africa by a mechanism called "barotropic instability," sometimes called horizontal shear instability.

Easterly wave formation may be visualized as the African easterly jet (with mean zonal velocity $\bar{u}$) breaking down into perturbations which develop rotation.

In dynamics, under barotropic, nondivergent conditions without friction, absolute vorticity $(\zeta + f)$ is conserved

$$\frac{D(\zeta+f)}{Dt} = \frac{\partial(\zeta+f)}{\partial t} + u\frac{\partial(\zeta+f)}{\partial x} + v\frac{\partial(\zeta+f)}{\partial y} = 0 \qquad (1)$$

ζ = relative vorticity $\equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$f \equiv 2\Omega \sin\theta$, $\theta \equiv$ latitude

$u \equiv$ zonal velocity

$v \equiv$ meridional velocity

Note that, $\frac{\partial f}{\partial t} = 0$, $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} \equiv \beta \equiv$ Beta effect

$$(2)\quad \frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + v\beta = 0$$

We want to investigate the stability of a horizontally sheared atmosphere for a given mean easterly current $\bar{u}(y)$. By defining mean and perturbation variables

$$\zeta = \bar{\zeta}(y) + \zeta'$$
$$u = \bar{u}(y) + u'$$
$$v = \underset{0}{\cancel{\bar{v}}} + v'$$

$$(3)\quad \underset{0}{\cancel{\frac{\partial \bar{\zeta}(y)}{\partial t}}} + \frac{\partial \zeta'}{\partial t} + (\bar{u}(y) + u')\frac{\partial}{\partial x}(\bar{\zeta}(y) + \zeta') + v'\frac{\partial(\bar{\zeta}(y) + \zeta')}{\partial y} + v'\beta = 0$$

We "linearize" this equation by neglecting the products of primes:

$$(4)\quad \frac{\partial \zeta'}{\partial t} + (\bar{u}(y) + u')\underset{0}{\cancel{\frac{\partial \bar{\zeta}}{\partial x}}} + \bar{u}(y)\zeta' + \underset{\text{neglect}}{\cancel{u'\zeta'}} + v'\frac{\partial \bar{\zeta}(y)}{\partial y} + \underset{\text{neglect}}{\cancel{v'\frac{\partial \zeta'}{\partial y}}} + v'\beta = 0$$

And investigate the conditions under which ζ' will grow under linear conditions.

$$(5)\quad \frac{\partial \zeta'}{\partial t} + \bar{u}(y)\zeta' + v'\frac{\partial \bar{\zeta}(y)}{\partial y} + v'\beta = 0$$

The continuity equation ~~under there~~ in a barotropic atmosphere is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, which may be solved by defining a streamfunction ψ through which:

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}, \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \nabla^2 \psi \tag{6}$$

Therefore, plugging (6) into (5) and noting that $\bar{\zeta} = -\frac{\partial \bar{u}}{\partial y}$ (since $\frac{\partial \bar{v}}{\partial x} = 0$) and $\psi' \equiv$ perturbation streamfunction

$$\frac{\partial}{\partial t}(\nabla^2 \psi') + \bar{u}(y)\nabla^2 \psi' + \left(\beta - \frac{\partial^2 \bar{u}(y)}{\partial y^2}\right) v = 0 \tag{7}$$

One then asks how a perturbation of finite amplitude $\phi(y)$ might grow if it had the wave solution:

$$\psi' = \phi(y)\, e^{-i\alpha(x-ct)} \tag{8}$$

Substitution of (8) into (7) gives after much algebra

$$(\bar{u}(y) - c)\left[\frac{\partial^2 \phi(y)}{\partial y^2} - \alpha^2 \phi(y)\right] + \underbrace{\left[\beta - \frac{\partial^2 \bar{u}(y)}{\partial y^2}\right]}_{A} \phi(y) = 0 \tag{9}$$

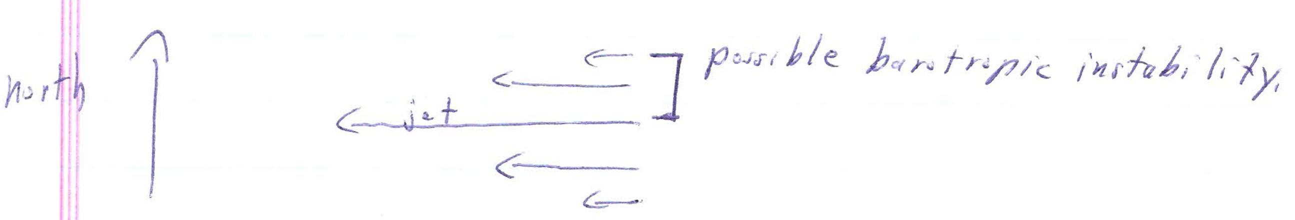
Where c is the phase speed of the wave and α is the wave number. From differential equations, we know that $\phi(y)$ will grow if the term A goes negative.

A necessary (but not sufficient) condition for growth of the perturbation is that:

(10) $$\left(\frac{\partial^2 \bar{u}(y)}{\partial y^2} - \beta \right) = +\frac{\partial}{\partial y}\left(\frac{\partial \bar{u}(y)}{\partial y} - f \right) = -\frac{\partial}{\partial y}(\bar{\xi}+f) = 0$$

This equation states that the meridional gradient of absolute vorticity must change sign in order for horizontal shear to initiate a wave perturbation. In other words, the north/south profile of $(\xi+f)$ must reach a maximum at some latitude.

The condition for barotropic instability is met between 10° and 20° N over Africa. This is because at 10-15° N there is a strong easterly jet due to the hot desert further north. North of the jet, the easterlies decrease rapidly, and $\frac{\partial \bar{u}}{\partial y} >> 0$. Therefore, $\frac{\partial u}{\partial y} >> f$ $(\xi+f<0)$, ∴ $\xi << 0$.



Under these conditions, the jet will break down into vortices which propagate westward and grow over Africa.

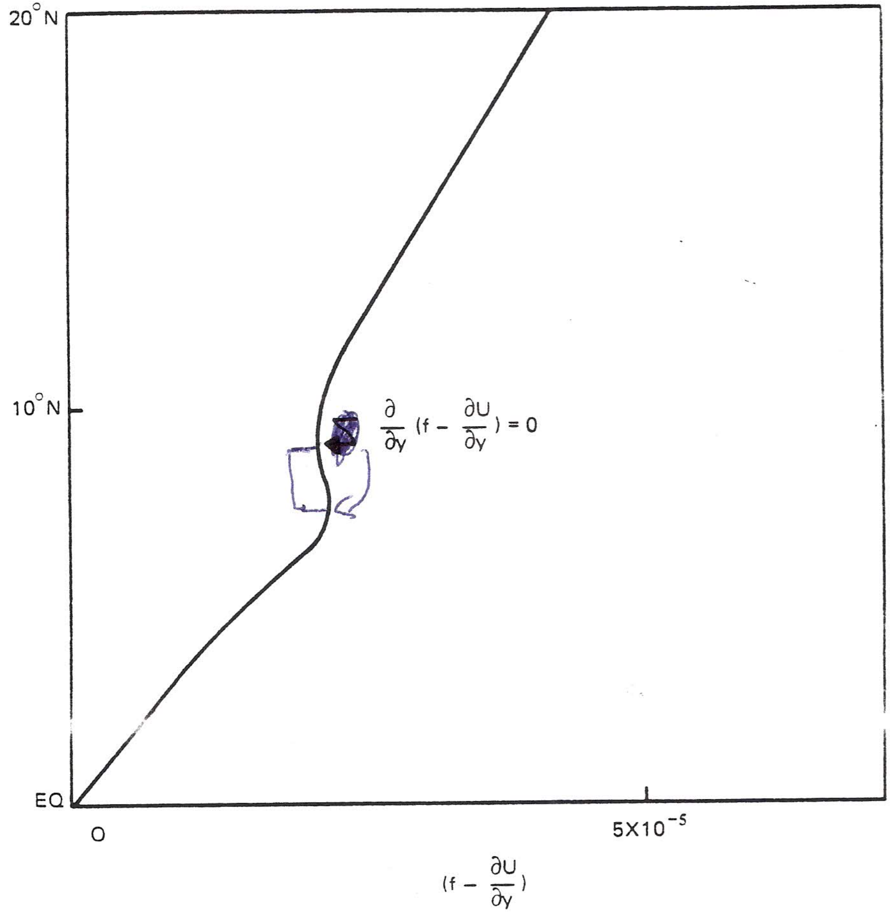
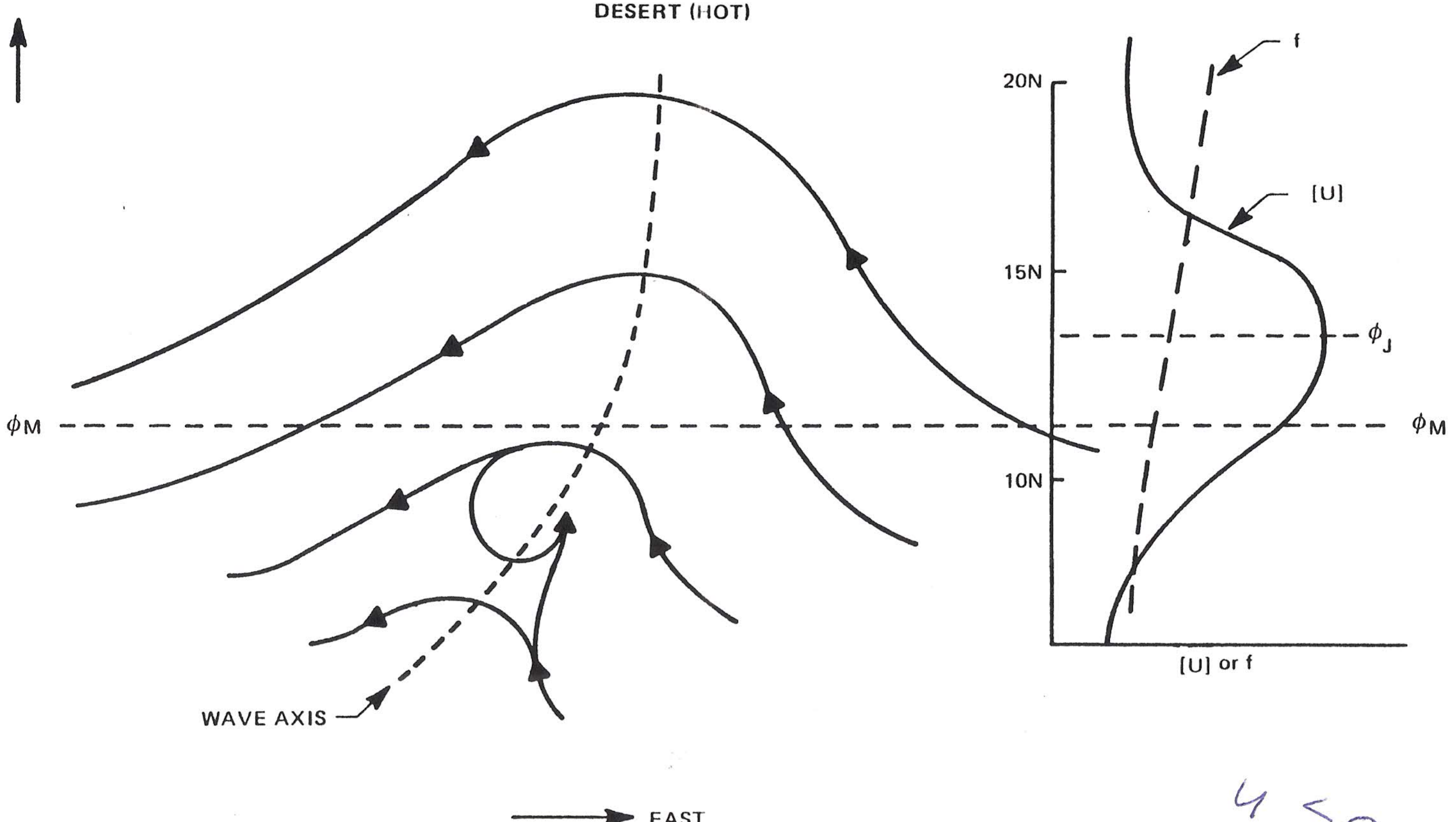


FIGURE 15.6 ABSOLUTE VORTICITY (sec^{-1}) CORRESPONDING TO ZONAL CURRENT IN FIGURE 15.5.



u < 0

FIGURE 15.13 SUMMER: EASTERLY WAVE OVER AFRICA
ϕ_M = LATITUDE OF MAX DISTURBANCE STRENGTH
ϕ_J = LATITUDE OF MID LEVEL EASTERLY JET (MILES)

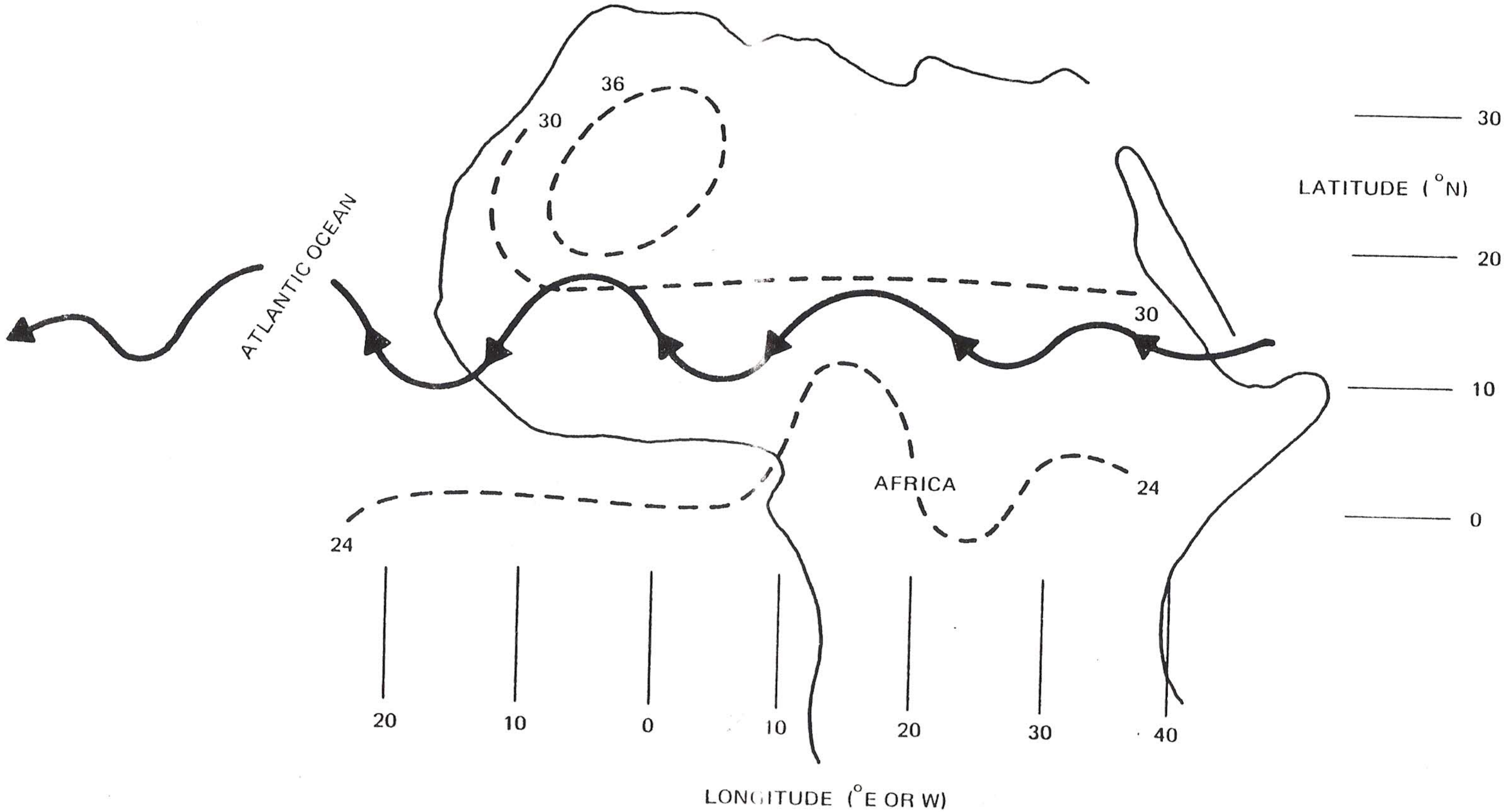


FIGURE 15.16 SCHEMATIC DEPICTION OF MID-LEVEL EASTERLY JET BREAKDOWN OVER AFRICA (HEAVY BARBED STREAMLINE) SHOWING INCREASE IN AMPLITUDE OF THE PERTURBATION AS IT MOVES WESTWARD FROM EAST AFRICA AND A DECREASE WESTWARD OF THE PERTURBATION OVER THE OCEAN. THE DASHED LINES ARE MEAN SURFACE ISOTHERMS (°C) IN JULY.

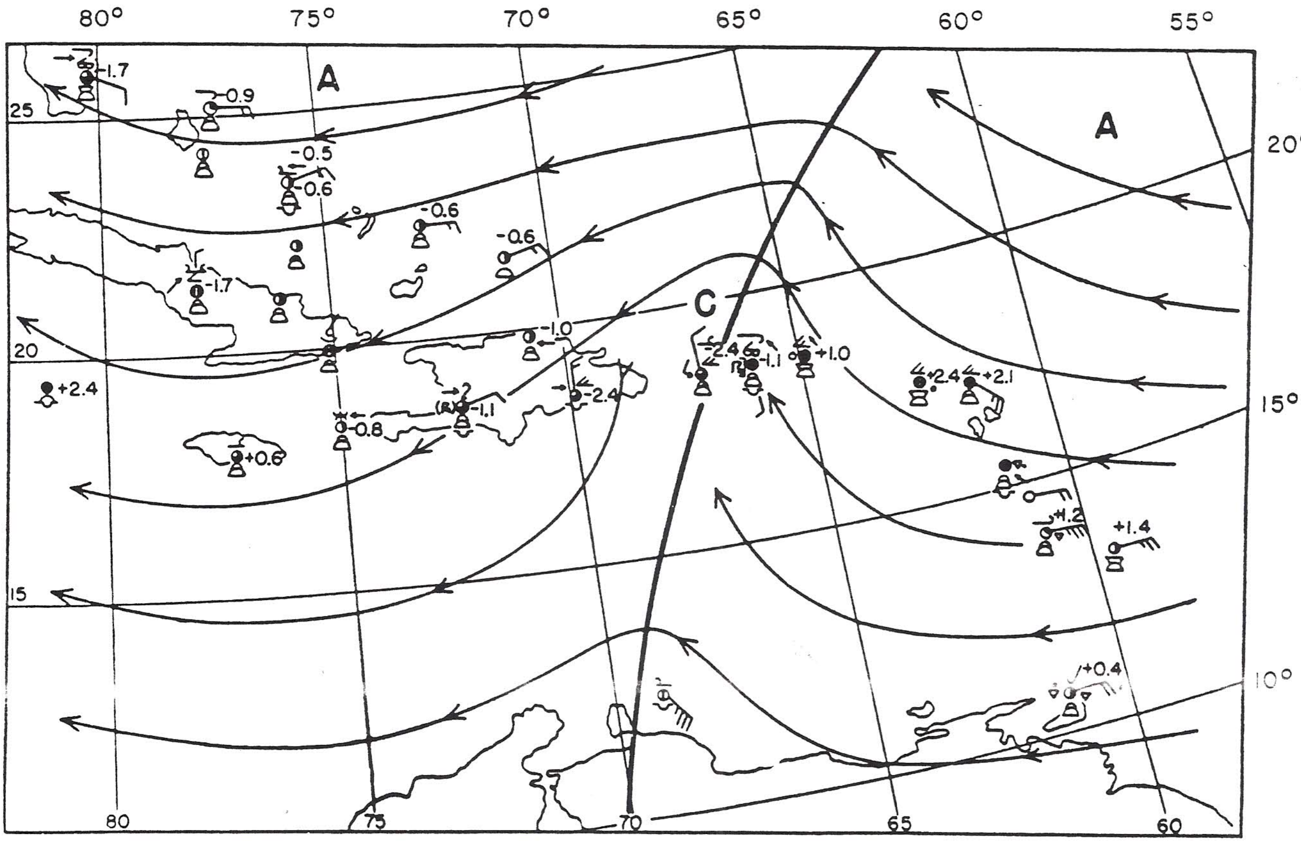


FIGURE 14.1 5,000-FOOT WINDS, SURFACE 24-HOUR PRESSURE CHANGES AND WEATHER REPORTS FOR THE CARIBBEAN AREA, JULY 12, 1944 (RIEHL, 1954)