8.2 Integration

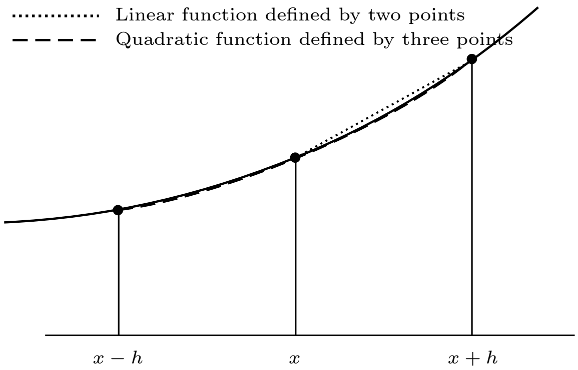

Figure 8-2. Interpolating functions for integration in terms of function values at $x-h$, x, and $x+h$

Figure 8-2 shows two different approximations for integrating the function by finding the area between the function and the x axis. The first is the *trapezoidal rule*, which consists of taking the function value at two points, fitting a straight line between them (the dotted line on the figure) and finding the area of that trapezium. It is

$$\int_{x}^{x+h} f(x)\,dx \approx \frac{h}{2}\left(f(x)+f(x+h)\right). \qquad \text{(Trapezoidal rule)}$$

On the other hand, if three points are used, a quadratic function can be used to interpolate the three, shown dashed in the figure, and the area under the quadratic obtained. The result is *Simpson's rule*

$$\int_{x-h}^{x+h} f(x)\,dx \approx \frac{h}{3}\left(f(x-h)+4f(x)+f(x+h)\right). \qquad \text{(Simpson's rule)}$$

In fact this result is also exact for a cubic function, and the accuracy of Simpson's rule goes like $O(h^3)$, which is surprisingly accurate.

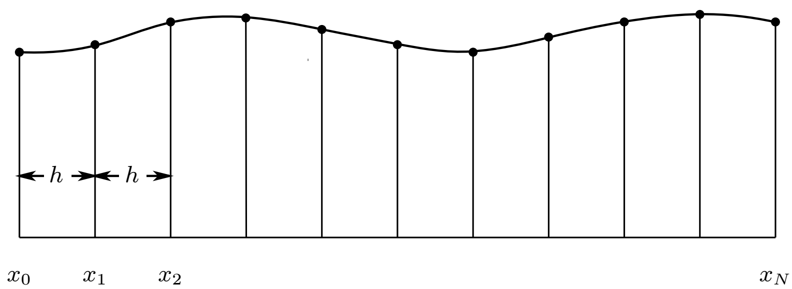

Figure 8-3. Typical subdivision for numerical integration

Of course in practice we need to find the area over a finite range, and so we use compound versions of these, as shown in Figure 8-3. The trapezoidal rule can be written

$$\int_{x_0}^{x_N} f(x)\,dx \approx \frac{h}{2}\left(f(x_0)+f(x_N)+2\sum_{i=1}^{n-1} f(x_i)\right). \qquad \text{(Compound trapezoidal rule)}$$

The compound Simpson's rule can only be applied for even numbers of panels (odd numbers of points, from 0 to N, such that N is an even number). It is

$$\int_{x_0}^{x_N} f(x)\,dx \approx \frac{h}{3}\left(f(x_0)+f(x_N)+4\sum_{i=1,\text{ odd}}^{N-1} f(x_i)+2\sum_{i=2,\text{ even}}^{N-2} f(x_i)\right). \qquad \text{(Compound Simpson's rule)}$$