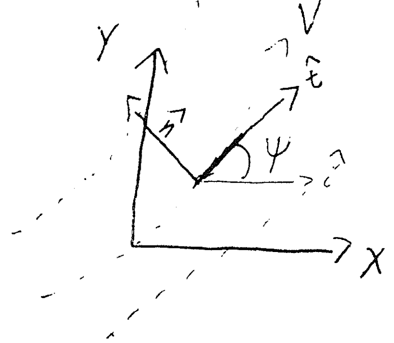
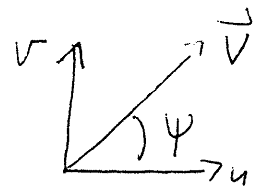
Horizontal Divergence in Natural Coordinates

Consider a set of streamlines with a horizontally varying windspeed field (implying contours of isotachs, but not shown to avoid messier drawing).



The vector $\vec{V}$ may be broken into its components u and v



$$u = |\vec{V}| \cos\psi$$
$$v = |\vec{V}| \sin\psi$$

Therefore

$$\frac{\partial u}{\partial x} = \cos\psi \frac{\partial |\vec{V}|}{\partial x} - |\vec{V}| \sin\psi \frac{\partial \psi}{\partial x}$$

$$\frac{\partial v}{\partial y} = \sin\psi \frac{\partial |\vec{V}|}{\partial y} + |\vec{V}| \cos\psi \frac{\partial \psi}{\partial y}$$

Now rotate x, y axes so that $\psi \to 0$. Then $\cos\psi \to 1$ and $\sin\psi \to 0$. Also $x \to s$ and $y \to n$

(22) $$\boxed{\nabla_h \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \underbrace{\frac{\partial |\vec{V}|}{\partial s}}_{\text{Stretching term}} + \underbrace{|\vec{V}| \frac{\partial \psi}{\partial n}}_{\text{Diffluence term}}}$$;

Stretching => speed divergence

Diffluence => Directional Divergence

Recall that $\frac{1}{R} = K = \frac{d\psi}{ds}$ (Eq ⑤).

Likewise, we can define a "radius of curvature" for the line normal to $s(x,y,t)$

$$\frac{1}{R_N} = K_N = \frac{d\psi}{dn} \tag{23}$$

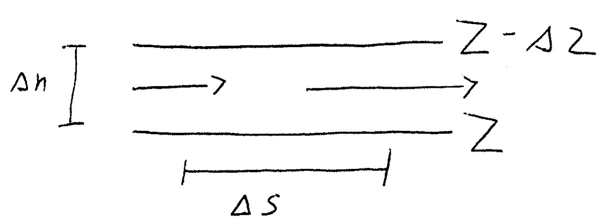
Hence, horizontal divergence may be written

$$\boxed{\nabla_h \cdot \vec{V} = \frac{\partial |\vec{V}|}{\partial s} + \frac{|\vec{V}|}{R_N}} \tag{24}$$

Interpretation :

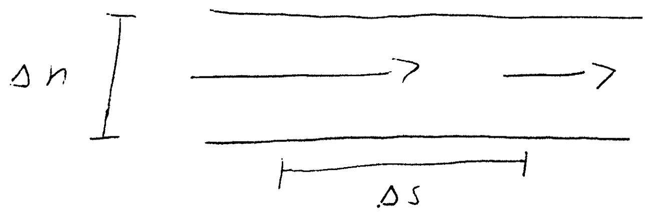
The stretching term measures the change in $|\vec{V}|$ along $s(x,y,t)$. Practically speaking, we access how $|\vec{V}|$ is changing along streamlines or geopotential heights.

Hypothetical Example 1; Δn = constant, but $|\vec{V}|$ changes downstream



$\frac{\partial |\vec{V}|}{\partial s} > 0$, since $|\vec{V}|$ increases downstream, the atmosphere is being "stretched." More air is being removed than added. This is speed divergence.

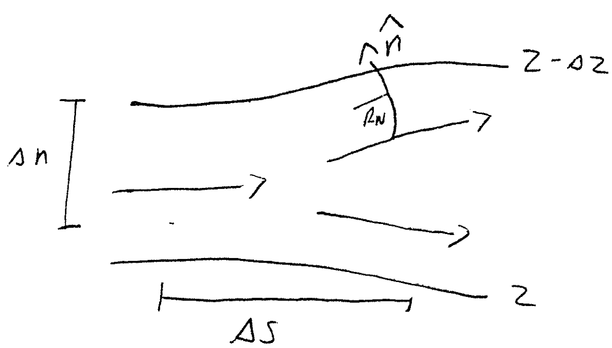
Likewise



$\frac{d|\vec{V}|}{ds} < 0$, speed convergence

Often, speed divergence and convergence is accessed by looking at isotach field.

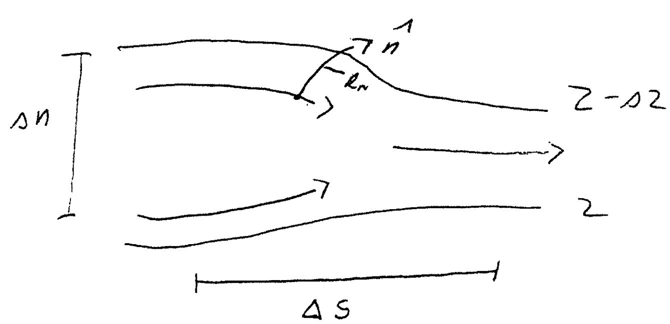
Example 2; ΔS = constant, but Δn changes downstream with $|\vec{V}|$ = constant.



Since $R_N > 0$; directional divergence, also called diffluence.

Air is "spreading out" at constant speed.

Likewise



$R_N < 0$

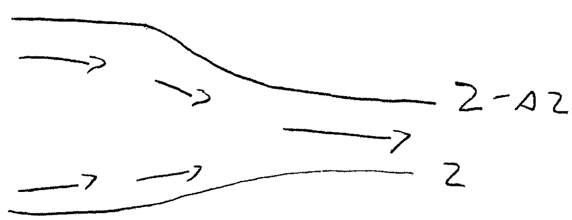
directional convergence, also called confluence.

NOTE! confluence is not total convergence!

To see if $\nabla_h \cdot \vec{V} > 0$ or $\nabla_h \cdot \vec{V} < 0$, need to consider speed divergence (convergence) and diffluence (confluence) together.

Furthermore, terms tend to cancel, making qualitative assessment difficult. Also, $\nabla_h \cdot \vec{V}$ is sensitive to measurement errors, making it difficult to calculate.

Example 3:



We have directional convergence, but speed divergence! This is typical. If the pressure gradient tightens downstream, that's associated with $\frac{\partial |\vec{V}|}{\partial s} > 0$ and confluence. So what is $\nabla_h \cdot \vec{V}$?

Determining which is dominating can be hard.

This is why divergence is a small term.

$\nabla_h \cdot \vec{V} \sim$ Order of $10^{-6} s^{-1}$