Dry adiabatic overview

Recall the following from the technical notes:

[snip]

where $c_p = 1004$ J deg^{-1} kg^{-1} is the specific heat at constant pressure. By inserting the hydrostatic equation $dp = -\rho g dz$:

$$dh = c_p dT + g dz \qquad 15$$

and for an adiabatic process where $dh = 0$, Eq (15) may be solved to compute the rate of temperature change for an unsaturated ("dry") air parcel undergoing ascent or descent $[(dT/dz)_d]$ as:

$$\Gamma_d = -\left(\frac{dT}{dz}\right)_d = \frac{g}{c_p} \qquad 16$$

where Γ_d is the *dry adiabatic lapse rate*, and is defined to be a positive quantity. The value of Γ_d is 9.8 deg km^{-1}. This says that an unsaturated air parcel will experience a temperature decrease (increase) of about 10°C per 1 km ascent (descent). This is often referred to as "dry adiabatic" ascent or descent.

[end snip]

Dry adiabatic lapse rate in units of feet

1 km = 3280 ft

1000 ft = (1 km/3280 ft)(1000 ft)=0.3 km

5000 ft = (1 km/3280 ft)(5000 ft)=1.5 km

Adiabatic temperature change per 1000 ft = (0.3 km)(9.8°C/km)≈3.0°C per 1000 ft

Adiabatic temperature change per 5000 ft = (1.5 km)(9.8°C/km)≈15.0°C per 5000 ft

Example for comparison to SkewT exercise

Initial parcel: p=845 mb=5000 ft; T=23°C

Lift parcel to p=705 mb=10000 ft; T=?

SkewT shows roughly 8°C when lifted dry adiabatically from p=845 mb (5000 ft) to p=705 mb (10000 ft).

If we calculate using 15.0°C per 5000 ft, T=23-15=8°C, which matches the chart.

Dry adiabatic lapse rate in pressure coordinates

Using the hydrostatic equation $dp = \rho g dz$, then from $\frac{\partial T}{\partial z} = -\rho g \frac{\partial T}{\partial p}$, we get $\left(\frac{\partial T}{\partial p}\right)_d = -\frac{\Gamma_D}{\rho g} = -\frac{1}{\rho c_p}$. From the ideal gas law, air density in the atmosphere decreases with temperature, pressure, and relative humidity. A good density calculator which accounts for all three effects is at: http://barani.biz/apps/air-density/ . It's essentially the ideal gas law with corrections for vapor pressure, written as $\rho = \left(\frac{p_{total}}{R_D T}\right)\left(1 - \frac{0.378e}{p_{total}}\right)$ where p_{total} includes both vapor pressure and dry air pressure, e is vapor pressure, and R_D is the dry gas constant. Assuming one is at sea level, the following table shows these variations:

Air density (kg m^{-3})			
For p=1000 mb	10°C	20°C	30°C
RH=100%	1.225	1.178	1.131
RH=75%	1.226	1.180	1.135
RH=50%	1.227	1.183	1.140
RH=25%	1.229	1.186	1.145
Air density (kg m^{-3})			
For p=950 mb	10°C	20°C	30°C
RH=100%	1.163	1.118	1.073
RH=75%	1.165	1.121	1.078
RH=50%	1.166	1.124	1.082
RH=25%	1.167	1.126	1.087

For simplicity, we'll assume a constant density using the online calculator. For an initial p=845 mb, 50% relative humidity, and T=23°C, the calculator gives ρ=0.988 kg m^{-3}. Converting pressure to Pascals (1 mb=100 Pa), temperature undergoing adiabatic expansion from 845 mb to 705 mb is 1/[(0.988 kg m^{-3})(1004 m^2 s^{-2} °C^{-1})] =14.1°C. So, this "back of the envelope" approximation is close.

For more accurate relationships of pressure change influence on temperature which avoids the density ambiguities, its best to use the Poisson relationship which includes the ideal gas law in the derivation (see any thermodynamics class). Then the relationship is:

$$T_1(p_1) = T_0(p_0)\left[\frac{p_1}{p_0}\right]^{\frac{R}{C_p}}$$

Recalling that temperature needs to be in Kelvin, and pressure in Pascals (units mb would work, too, since it's a ratio):

$$T_1 = (23 + 273.15)\left[\frac{70500}{84500}\right]^{\frac{287}{1004}} = 281.20°\text{K}$$

So the temperature at 705 mb is 281.2-273.15=8.05°C. Correct!

Thought question: in a major hurricane over the ocean where central pressure is 900 to 960 mb, air parcels should cool flowing towards the storm due to adiabatic expansion. But the surface air temperature roughly stays the same. What's happening? What is special about the hurricane boundary layer over the ocean?

Potential temperature

Potential temperature θ is the temperature an unsaturated parcel of air would have if brought dry adiabatically from its initial location to a reference level. The reference level in meteorology is 1000 mb, but in oceanography can be different levels of the ocean and dependent on its usage.

Mathematically, potential temperature is simply an extension of the Poisson equation, where p_{ref} is a reference level.

$$\theta = T_0(p_0)\left[\frac{p_{ref}}{p_0}\right]^{\frac{R}{C_p}}$$

The potential temperature for the air parcel at p=845 mb; T=23°C; is

$$\theta = (23 + 273.15)\left[\frac{100000}{84500}\right]^{\frac{287}{1004}} = 310.7°K$$

So the temperature at 1000 mb is 310.7-273.15=37.6°C. Close to the SkewT, as it should be!