

Name: _____

Assignment Dynamic meteorology

Directions: Complete *neatly* on separate sheets of paper. Unorganized and unreadable answers will result in lost points. Show all steps, or little to no point credit will be given. ~~Due Monday, August 25 by 11 AM.~~

- 8) 1) Given that $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 4\hat{k}$, compute:
- $\vec{A} \cdot \vec{B}$
 - $\vec{A} \times \vec{B}$
 - $|\vec{A}|$
 - “normalized” \vec{A}
- 4) 2) Prove that $(\vec{A} \times \vec{B}) \cdot \vec{B} = 0$ for all vectors \vec{A} and \vec{B}
- 4) 3) Given that $\vec{A} = 3\hat{i} - 7\hat{j} + 2\hat{k}$ and $\vec{B} = 10\hat{i} + 4\hat{j} - \hat{k}$, compute $\vec{A} \cdot \vec{B}$. What does this answer suggest about the relationship between \vec{A} and \vec{B} ?
- 4) 4) Given that $\vec{A} = 4\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = -2\hat{i} + 3\hat{j} - \hat{k}$, compute $\vec{A} \times \vec{B}$. What does this answer suggest about the relationship between \vec{A} and \vec{B} ?
- 2) 5) Given that $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$, write a generic expression for ~~divergence~~ $\nabla \cdot \vec{V}$.
- 4) 6) On graph paper, draw the vector field for $\vec{V}_H = -x\hat{i} - y\hat{j}$. Does this represent divergence, convergence, or neither?
- 8) 7) Write the Taylor Series expansion for $f(x) = x^{1/2}$ at x_0 out to the third derivative. Compute $(4.1)^{1/2}$ where $x_0 = 4$ and compare it to the true solution.
- 4) 8) Let $x_0 = 0$, and write out a Taylor series expansion of e^x to the “nth” derivative. This approximation of e^x is used in many physics and meteorological formulations.
- 4) 9) Compute the derivative of $f(x) = 2x^3 - 5x + 4$ by:
- Using the formal definition in which ~~the limit~~ $\lim_{\Delta x \rightarrow 0}$
 - Using the shortcut $\frac{d}{dx}(x^n) = nx^{n-1}$ ~~the shortcut~~
- 2) 10) From problem 9, compute the slope of $f(x)$ at $x = 4$.
- 6) 11) Approximate the slope computation in problem 10 at $x = 4$ by using “centered finite differencing” for the following three formulas:

$$\frac{f(x = 4.5) - f(x = 3.5)}{\Delta x}$$

where $\Delta x = 4.5 - 3.5 = 1$

$$\frac{f(x = 4.25) - f(x = 3.75)}{\Delta x}$$

where $\Delta x = 4.25 - 3.75 = 0.5$

$$\frac{f(x = 4.1) - f(x = 3.9)}{\Delta x}$$

where $\Delta x = 4.1 - 3.9 = 0.2$

Compare and discuss these results with those found in problem 10.

4 12) Given $f(x, y) = x^2y^3$, find: a) $\partial f/\partial x$, b) $\partial f/\partial y$.

4 13) Given $f(x, y) = e^{2x} \ln(x^2 + y^2 + 1)$, find: a) $\partial f/\partial x$, b) $\partial f/\partial y$.