Filter analysis

$$\phi_i^{n+1} = \phi_i^n + a\left(\phi_{i+1}^n + \phi_{i-1}^n - 2\phi_i^n\right)$$

Analysis gives

$$\lambda e^{i\omega_r \delta t} = 1 + a\left(e^{ik\delta x} + e^{-ik\delta x} - 2\right)$$

$$\lambda\left(\cos(\omega_r \delta t) + i\sin(\omega_r \delta t)\right) = 1 + a\left(2\cos(k\delta x) - 2\right)$$

$$\lambda \cos(\omega_r \delta t) = 1 + 2a\left(\cos(k\delta x) - 1\right)$$

$$\lambda \sin(\omega_r \delta t) = 0$$

2nd implies $\omega_r = 0$ and no propagation ; Therefore

$$\boxed{\lambda = 1 + 2a\left(\cos\left(\frac{2\pi}{n}\right) - 1\right)}$$