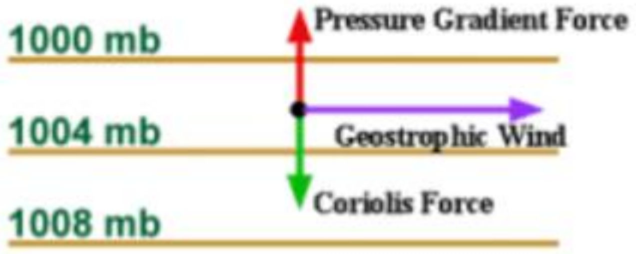
Geostrophic wind

Above the boundary layer where friction is negligible, isobars or height lines typically show winds parallel to the contour lines. In straight flow, there is a close balance between the Coriolis force and the pressure gradient force. This is known as geostrophic balance, as depicted on the right.



Deviations from geostrophic balance occur in regions where there is ascent or descent. That is a discussion to be elaborated on in synoptic class.

The frictionless horizontal equations of motion in (x,y,z) coordinates are written as:

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \qquad \frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu \qquad f = 2\Omega \sin\phi$$

where f is the Coriolis parameter and the advection terms are imbedded within the total derivatives $\frac{Du}{Dt}$ and $\frac{Dv}{Dt}$

The 2D (horizontal) vector form of the equations of motion is:

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla p - f\hat{k} \times \vec{V}$$

When the two forces balance, the acceleration is zero:

$$\cancelto{0}{\frac{Du}{Dt}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \qquad \cancelto{0}{\frac{Dv}{Dt}} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

Solve for the wind component:

$$v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x} \qquad u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y}$$

where the subscript g indicates geostrophic. Note that the geostrophic wind is proportional to the pressure gradient. Also note that due an inversely proportionality to latitude (in the f term), the same pressure gradient yields lower wind magnitudes at higher latitudes. This is because balance is achieve sooner at higher latitudes.

The vector version of the geostrophic wind is:

$$\vec{V}_g = \hat{k} \times \frac{1}{f\rho}\nabla p$$

Which indicates the wind direction is parallel to isobars (Remember the right-hand rule)

In (x,y,p) coordinates, these become

$$\cancelto{0}{\frac{Du}{Dt}} = -\frac{\partial \Phi}{\partial x} + fv \qquad \cancelto{0}{\frac{Dv}{Dt}} = -\frac{\partial \Phi}{\partial y} - fu$$

Or:

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \qquad u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y}$$

And the vector version of the geostrophic wind is:

$$\vec{V}_g = \hat{k} \times \frac{1}{f} \nabla \Phi$$

which indicates the wind direction is parallel to isohypses (height lines)

The Rossby number

The Rossby number is the ratio of the inertial acceleration term to the Coriolis term:

$$R_o = \frac{\left| {D\vec{V}}/{Dt} \right|}{\left| f\hat{k} \times \vec{V} \right|}$$

The inertial acceleration term scales as $\frac{U^2}{L}$. The Coriolis term scales as *fU*.

$$R_o = \frac{U^2/L}{fU} = \frac{U}{fL}$$

When $R_o \ll 1$, then the acceleration term is small, and the only two terms with any magnitude are the pressure gradient terms and the Coriolis term, which must be nearly in balance. Hence, small (<<1) Rossby numbers indicate geostrophic flow is dominant.

Note that for synoptic scales at mid-latitudes where $\mathrm{U} \sim 10$ ms^{-1}, $\mathrm{f} \sim 0.0001$ s^{-1}, and $\mathrm{L} \sim 1{,}000{,}000$ m, R_o=0.1.