Gravitational Force

Gravitation, first discovered by Newton, is the universal force attracting any two bodies towards each other. Gravitation increases as the mass of either (or both) increases. Gravitation is also inversely proportional to the distance squared. In terms of the earth's gravitation $\vec{g}^*$, the two bodies attracting each other are the earth and the atmosphere, and the distance *(r)* is the mean radius of the earth *(a)* and the height of the atmosphere (*z*), or $r = a + z$. At sea level where $z = 0$, gravitation is equal to the universal constant 9.8 ms^{-2}.

Since gravitation decreases as the distance between two --- in this case, the earth and the atmosphere --- increases, $\vec{g}^*$ must decrease with height. However, since $z \langle\langle a$, the decrease is negligible for most meteorological purposes. For example, at 10 km the decrease is 0.3%. Therefore, $\vec{g}^*$ is simply treated as a constant. See pg 7-8 in Holton for details.

Molecular and Eddy Viscous Forces

Viscosity is the resistance to flow of a liquid or gas (i.e., molasses has a low viscosity and motor oil has a high viscosity). For the atmosphere below 100 km, molecular viscosity υ is only important within a few centimeter's of the earth's surface. At the ground, air motion is zero, but within a few centimeters air motion increases and viscosity transports air molecules at this level, inducing a *shearing stress* $\tau_{zx} = \mu \frac{\partial u}{\partial z}$ (where μ is the *dynamic viscosity coefficient* and considering only the x direction). The net result is a "mixing" of momentum between these two levels.

In a manner analogous to the pressure gradient derivation, Holton shows that the x-direction molecular viscous force $F_{rx}(mol)$ may be written for constant μ:

$$F_{rx}(mol) = \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) = \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

where $\upsilon = \mu / \rho$ is the *kinematic viscosity coefficient.* This may be rewritten as:

$$F_{rx}(mol) = \upsilon \nabla^2 u$$

where ∇^2 is a Laplacian term, and generally acts to diffuse the quantity in question. Again, molecular friction is only important in the first few centimeters, a region called the "viscous sublayer."

The "friction" of concern to meteorologists is due to mixing by turbulent eddies within 1 km of the surface, a region called the "planetary boundary layer." This mixing of different eddies with a size of several hundred meters decelerates wind flow, and is analogous to the molecular shearing stress in the viscous sublayer. This analogy is called the “mixing length hypothesis,” and is ~~will be~~ discussed more in Chapter 8 n fact, this subject is so complex a whole field, called *boundary layer meteorology*, is devoted to it, and there is no consensus on how friction caused by turbulent eddies should be handled.

One theory states that the x-direction eddy viscous force $F_{rx}(eddy)$ may be written as:

$$F_{rx}(eddy) = K_M \nabla^2 u$$

where K_M is the *eddy viscosity*. Further discussions of eddy friction will occur in Chapter 5. For the first 4 chapters, friction will simply be written as $\vec{F}_r$ without any more details.

Read pg ~~8-11~~ for more details on this topic.

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