Name____________________________

# Filter homework, hand exercises

1) The following are average monthly temperatures (°C) in Ahmadabad, India, a city impacted by the annual south Asian monsoon

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 20.3 | 22.8 | 27.1 | 31.3 | 33.5 | 32.7 | 29.4 | 28.3 | 28.7 | 28.4 | 24.5 | 21.1 |

Compute the average temperature, given that (3 pts)

$\bar{T} = \frac{1}{12}\sum_i T_i$

$$\bar{T} = \frac{1}{12}(20.3 + 22.8 + 27.1 + 31.3 + 33.5 + 32.7 + 29.4 + 28.3 + 28.7 + 28.4 + 24.5 + 21.1)$$

$$\bar{T} = \frac{1}{12}(348.4)$$

$$\bar{T} =$$

Compute amplitude coefficients for n=1. Note that the length of the domain is the last time value at t=12. The fact that the index is also 12 is just a coincidence. Usually the length of the domain is not the same as the number of values. (6 pts)

| i | t | $T_i$ | $\sin(2\pi 1t/12)$ | $\cos(2\pi 1t/12)$ | $T_i sin(2\pi 1t\ /\ 12)$ | $T_i cos(2\pi 1t\ /\ 12)$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 20.3 | 0.5 | 0.866 | 10.15 | 17.58 |
| 2 | 2 | 22.8 | 0.866 | 0.5 | 19.75 | 11.4 |
| 3 | 3 | 27.1 | 1.0 | 0 | 27.1 | 0 |
| 4 | 4 | 31.3 | 0.866 | -0.5 | 27.11 | -15.65 |
| 5 | 5 | 33.5 | 0.5 | -0.866 | 16.75 | -29.01 |
| 6 | 6 | 32.7 | 0 | -1 | 0 | -32.7 |
| 7 | 7 | 29.4 | -0.5 | -0.866 | -14.7 | -25.46 |
| 8 | 8 | 28.3 | -0.866 | -0.5 | -24.5 | -14.15 |
| 9 | 9 | 28.7 | -1.0 | 0 | -28.7 | 0 |
| 10 | 10 | 28.4 | -0.866 | 0.5 | -24.59 | 14.2 |
| 11 | 11 | 24.5 | -0.5 | 0.866 | -12.25 | 21.22 |
| 12 | 12 | 21.1 | 0 | 1 | 0 | 21.1 |
| | | | | | | |
| Total | | | | | -31.47 | -3.9 |

$A_1 = \frac{2}{12}\sum_i T_i \sin\frac{2\pi 1t}{12}$; $B_1 = \frac{2}{12}\sum_i T_i \cos\frac{2\pi 1t}{12}$

$A_1 =$ $B_1 =$

Compute amplitude coefficients for n=2. (12 pts)

| i | t | $T_i$ | $\sin(2\pi 2t/12)$ | $\cos(2\pi 2t/12)$ | $T_i sin(2\pi 2t\ /\ 12)$ | $T_i cos(2\pi 2t\ /\ 12)$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 20.3 | 0.866 | 0.5 | 10.15 | 17.58 |
| 2 | 2 | 22.8 | 0.866 | -0.5 | -11.4 | 19.75 |
| 3 | 3 | 27.1 | 0 | -1 | -27.1 | 0 |
| 4 | 4 | 31.3 | -0.866 | -0.5 | -15.65 | -27.11 |
| 5 | 5 | 33.5 | -0.866 | 0.5 | 16.75 | -29.01 |
| 6 | 6 | 32.7 | 0 | 1 | 32.7 | 0 |
| 7 | 7 | 29.4 | 0.866 | 0.5 | 14.7 | 25.46 |
| 8 | 8 | 28.3 | 0.866 | -0.5 | -14.15 | 24.51 |
| 9 | 9 | 28.7 | 0 | -1 | -28.7 | 0 |
| 10 | 10 | 28.4 | | | | |
| 11 | 11 | 24.5 | | | | |
| 12 | 12 | 21.1 | | | | |
| | | | | | | |
| Total | | | | | | |

$A_2 = \frac{2}{12}\sum_i T_i \sin\frac{2\pi 2t}{12}$; $B_2 = \frac{2}{12}\sum_i T_i \cos\frac{2\pi 2t}{12}$

$A_2 =$ $B_2 =$

Compute amplitude coefficients for n=3. (9 pts)

| i | t | $T_i$ | $\sin(2\pi 3t/12)$ | $\cos(2\pi 3t/12)$ | $T_i sin(2\pi 3t\ /\ 12)$ | $T_i cos(2\pi 3t\ /\ 12)$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 20.3 | | | 0 | 20.3 |
| 2 | 2 | 22.8 | | | -22.8 | 0 |
| 3 | 3 | 27.1 | | | 0 | -27.1 |
| 4 | 4 | 31.3 | | | 31.3 | 0 |
| 5 | 5 | 33.5 | | | 0 | 33.5 |
| 6 | 6 | 32.7 | | | -32.7 | 0 |
| 7 | 7 | 29.4 | | | 0 | -29.4 |
| 8 | 8 | 28.3 | | | 28.3 | 0 |
| 9 | 9 | 28.7 | | | 0 | 28.7 |
| 10 | 10 | 28.4 | | | -28.4 | 0 |
| 11 | 11 | 24.5 | | | 0 | -24.5 |
| 12 | 12 | 21.1 | | | 21.1 | 0 |
| | | | | | | |
| Total | | | | | -3.2 | 1.5 |

$A_3 = \frac{2}{12}\sum_i T_i \sin\frac{2\pi 3t}{12}$; $B_3 = \frac{2}{12}\sum_i T_i \cos\frac{2\pi 3t}{12}$

$A_3 =$ $B_3 =$

At t=5, add up the three harmonics. How well does it compare to the May temperature? (6 pts)

Compute the amplitude of the three waves using $C_n = \sqrt{A_n^2 + B_n^2}$ (3 pts)

$C_1 =$

$C_2 =$

$C_3 =$

Compute the power of the three waves using $P_n = \frac{C_n^2}{2}$ (3 pts)

$P_1 =$

$P_2 =$

$P_3 =$

Given that the sample variance of the time series is $s^2$=18.88°$C^2$, compute the variance explained by each harmonic using $R_n^2 = \frac{i_{last} P_n}{(i_{last}-1)s^2}$ where $i_{last}$ is the number of i indexes (in this case, 12). The ratio of $i_{last}$ to $i_{last} - 1$ is required to compensate for the fact this is a limited data sample, and is often overlooked in class lectures and textbooks. Note: $R_n^2$ is sometimes called the normalized spectral density. (3 pts)

$R_1^2 =$

$R_2^2 =$

$R_3^2 =$

With this information, answer the following: (2 pts each)

a) What does the n=1 signal represent?

b) What does the n=2 signal represent?

c) Which signal explains the most variance?

d) How much total variance is explained by the first two harmonics?

2) Given that

$$\psi(x) = 4\sin\frac{2\pi 3x}{L} + 1.3\cos\frac{2\pi 3x}{L} + 9\sin\frac{2\pi 4x}{L}$$

Use the orthogonality relationships to compute (12 pts)

$A_2 =$

$B_2 =$

$A_3 =$

$B_3 =$

$A_4 =$

$B_4 =$

3) Given five months of data

| | | | | |
|---|---|---|---|---|
| 20.3 | 27.1 | 33.5 | 29.4 | 24.5 |

a. Write expressions for a centered three-month running mean (6 pts)

b. Write expressions for a 1-2-1 Shapiro Filter. Assume periodic boundary conditions (6 pts)

c. What is the main philosophical difference between a digital filter and a running mean? (6 pts)

d. What is a highpass filter? (3 pts)

e. What is a lowpass filter? (3 pts)

f. What type of equation is used to quantify the reduction of signal amplitude by wavelength and number of passes? (3 pts)

4) What is Gibbs phenomenon? (3 pts)

5) What filter is designed to minimize Gibbs phenomenon? (3 pts)

What math function is the basis for the filter? (3 pts)

6) What is a recursive filter? (6 pts)