Inertial Flow

Recall:

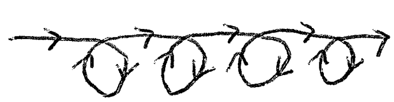
$$(1)\qquad \frac{D|\vec{V}|}{Dt} = -\frac{\partial \Phi}{\partial s} \qquad (1a)$$

$$\frac{|\vec{V}|^2}{R} + f|\vec{V}| = -\frac{\partial \Phi}{\partial n} \qquad (1b)$$

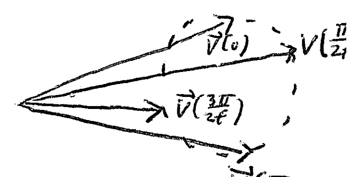
For the case where $\nabla \Phi = 0$ $\left(\frac{\partial \Phi}{\partial s} = 0, \frac{\partial \Phi}{\partial n} = 0\right)$, which implies $\frac{D|\vec{V}|}{Dt} = 0$ ($|\vec{V}| = \text{constant}$), (1) becomes

$$(2)\qquad \frac{|\vec{V}|^2}{R} + f|\vec{V}| = 0$$

Therefore, $R = -\frac{|\vec{V}|}{f}$. Since $|\vec{V}| = \text{constant}$, $f = \text{constant}$ (neglecting the latitudinal dependence of f), and because $R < 0$, a slowly moving an parcel will follow circular paths in an anticyclonic sense. Consider a parcel moving slowly east, then it will "loop clockwise" in the northern hemisphere (counterclockwise in southern hemisphere):



or following the motion



The circumference of a circle is:

$$(3)\qquad S = 2\pi|R| = 2\pi\frac{|\vec{V}|}{|f|}$$

The period of motion is given by P, defined as

$$(4)\qquad P = \frac{S}{|\vec{V}|}$$

Or, by substituting 3 into 4

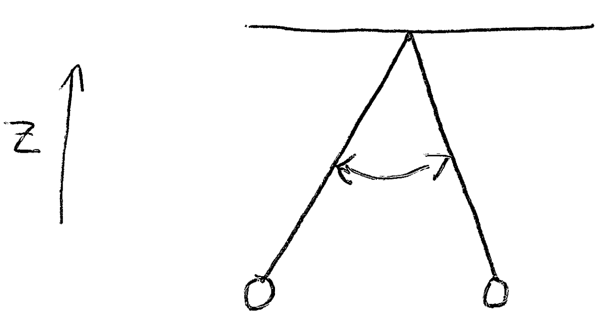
5 $$P = \frac{2\pi}{|f|}$$

Which means the period depends on the Coriolis parameter, not on wind speed. By substituting $f = 2\Omega \sin\phi$ with $\Omega = \frac{2\pi}{day} = \frac{\pi}{12\,h}$ (where h is "hour") into 5

6 $$P = \frac{\frac{1}{2}\,day}{|\sin\phi|} = \frac{12\,h}{|\sin\phi|}$$

P is the time required for a "Foucault pendulum" to turn through an angle of 180°. Hence, it is often referred to as one-half "pendulum day."

What is a Foucault pendulum? Consider a large, heavy ball attached to a strong but thin rope connected to a high ceiling. This is often seen at museums.



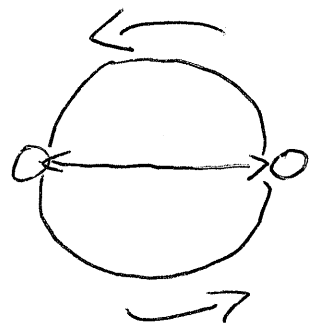
The ball is heavy enough that, when set in motion, friction is negligible.

Now suppose dominoes are placed on the ground which circle the pendulum. The swinging pendulum

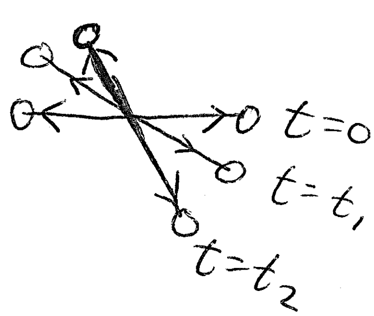
will knock over two dominoes at opposite ends of the circle. However, the pendulum will slowly swing to the right. A few minutes later, two more dominoes will fall. Eventually, after $\frac{1}{2}$ pendulum day, all the dominoes will be down.

What's happening here? In a sense, the earth is slowly turning under the pendulum. Let's take the simple case at the north pole where $P = 12\,h$.

Looking straight down on the north pole, it appears as



Or in our frame of reference (noninertial)



According to (6), P increases as one moves away from the poles. Fig 3.3 shows that the ocean experiences "inertial oscillations" at 13° latitude of 53 h.

Pure inertial oscillations are rarely found in the atmosphere because they are damped by friction, or because other forces also control the evolution of motion. In addition, inertial oscillations may interact with phenomena with different time scales and thus be masked. The atmosphere is also rather efficient at bringing "ageostrophic motion" back to geostrophic balance.

When do we see inertial oscillations?

1) Center of highs where $\nabla p = 0$.

2) The ocean, where pressure gradients can be weak. Sometimes in the Baltic Sea the water is strongly stratified in the vertical, which permits a nearly frictionless gliding of one layer over the next. Currents here at 14 meters turn continually to the right with a period of 14 hours under such conditions.

These same derivations may also be derived from the horizontal equations of motion in (x,y) coordinates. Under frictionless conditions where the pressure gradient vanishes:

$$(7) \quad \frac{du}{dt} = f\frac{dy}{dt} \quad ; \quad \frac{dv}{dt} = -f\frac{dx}{dt}$$

Upon integration of (7)

$$(8) \quad u = fy + A \quad ; \quad v = -fx + B$$

where A and B are constants of integration. Also

$$(9) \quad |\vec{V}|^2 = u^2 + v^2$$

Substitute (8) into (9)

$$(10) \quad \left[x - \left(\frac{B}{f}\right)\right]^2 + \left[y + \left(\frac{A}{f}\right)\right]^2 = \left(\frac{|\vec{V}|}{f}\right)^2$$

This is the equation of a circle $R = \frac{|\vec{V}|}{f}$ with its center at $X = \frac{B}{f}$ and $Y = -\frac{A}{f}$. X and Y define an "inertia circle."