Isobaric pressure gradient force

Recall gradient conversion from the *p* to *z* coordinate along x axis

$$\left(\frac{\partial q}{\partial x}\right)_p = \left(\frac{\partial q}{\partial x}\right)_z + \left(\frac{\partial q}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_p$$

Let *q*=*p* , then

$$\left(\frac{\partial p}{\partial x}\right)_p = \left(\frac{\partial p}{\partial x}\right)_z + \left(\frac{\partial p}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_p$$

Note that:

$$\left(\frac{\partial p}{\partial x}\right)_p = 0, \qquad \textit{pressure constant along an isobar}$$

$$\left(\frac{\partial p}{\partial z}\right)_x = -\rho g, \qquad \textit{substitute hydrostatic equation}$$

Then

$$0 = \left(\frac{\partial p}{\partial x}\right)_z - \rho g \left(\frac{\partial z}{\partial x}\right)_p = \left(\frac{\partial p}{\partial x}\right)_z - \rho \left(\frac{\partial \Phi}{\partial x}\right)_p, \qquad \textit{since } d\Phi = g dz \ \left(\textit{recall } \Phi = \int_0^z g\, dz\right)$$

Divide by density and rearrange

$$\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = \left(\frac{\partial \Phi}{\partial x}\right)_p$$

which relates the pressure gradient force on a height surface to the pressure gradient force on an isobar.

The advantage of an isobaric coordinate system is that the horizontal pressure gradient force, which is now only measured by the gradient of geopotential height at constant pressure, is that density is no longer a consideration.