(b) Leapfrog-in-time and centered-in-space schemes

The advection equation can also be approximated by the leapfrog (second-order centered) in time and second-order centered difference in space scheme,

$$\frac{u_i^{\tau+1} - u_i^{\tau-1}}{2\Delta t} = -c\frac{u_{i+1}^{\tau} - u_{i-1}^{\tau}}{2\Delta x}. \tag{12.3.44}$$

$f'(x)$ from (12.3.62) and (12.3.63).

Now apply the fourth-order centered difference scheme in space and the leapfrog scheme in time to the advection equation (12.3.1),

$$\frac{u_i^{\tau+1} - u_i^{\tau-1}}{2\Delta t} + c\left[\frac{8(u_{i+1}^{\tau} - u_{i-1}^{\tau}) - (u_{i+2}^{\tau} - u_{i-2}^{\tau})}{12\Delta x}\right] = 0. \tag{12.3.65}$$

Solve for $u_i^{\tau+1}$,

$$u_i^{\tau+1} = u_i^{\tau-1} - \frac{C}{6}\left[8(u_{i+1}^{\tau} - u_{i-1}^{\tau}) - (u_{i+2}^{\tau} - u_{i-2}^{\tau})\right], \tag{12.3.66}$$

where C is the *Courant number*, as defined earlier. It can be shown that

$$C = \frac{c\Delta t}{\Delta x}, \tag{12.3.11}$$

is called the *Courant number*. Substituting $\omega = \omega_r + i\omega_i$, where both ω_r and ω_i are real numbers, on the left side of (12.3.10), yields