See also Appendix J in Djuric

Mathematical properties of fronts

A front is a three-dimensional surface of discontinuity in density between two air masses. A cold front is defined as the instantaneous movement of cold air is such that it is replacing warmer air. With a warm front, warm air is replacing cold air instantaneously. On a synoptic map, if the wind component in the cold air perpendicular to the front is toward the warmer air, it is a cold front, and vice versa for a warm front.

Nomenclature concerning discontinuities

If we consider a property (let call it "a"), which is itself discontinuous, "a" is said to possess a <u>zero-order</u> discontinuity, since the zero-order derivative (the function itself) is discontinuous.

If the transition layer between the two air masses is sufficiently thin, a front approximates a true surface of discontinuity in density and may be defined as a discontinuity surface of zero-order.

Each line of discontinuity in a fluid must have certain boundary conditions. The first of these is the <u>dynamic boundary condition</u>. The dynamic boundary condition states that pressure must be continuous through an internal boundary (like a front); that is, the pressures immediately on either side of a discontinuity must be the same. This is because infinite pressure gradient forces are not possible.

The second boundary condition is the <u>kinematic boundary condition</u>. When applied to a discontinuity, this requirement means that the fluid properties immediately on either side of the internal boundary must have the same component of velocity perpendicular to the boundary. This is because an infinite velocity gradient would generate infinite divergence, which is not possible.

These boundary conditions apply to higher-order discontinuities as well. If property "a" is continuous, but its first derivative with respect to distance is discontinuous, then it is said to possess a <u>first-order discontinuity</u>. In general, a discontinuity in the <u>nth</u> derivative of a quantity is described as an <u>nth-order</u> discontinuity of that quantity.

Fronts as discontinuities

From a broad-scale perspective, a front approximates a true surface of discontinuity in density of zero-order. The dynamic boundary condition specifies pressure to be equal, hence the gas law ($p = \rho RT$) implies that a discontinuity in density is also a discontinuity of