Modeling

Introduction

The first attempts at numerical modeling were performed by Dr. Lewis Richardson in 1922, in which a forecast of surface pressure was attempted. The forecast, which involved tedious calculations by hand, failed miserably, with forecasts 30-40 mb off. This failure (due to using what we now know was a poor numerical algorithm) and the enormous computational time involved discouraged further research for several generations.

With the invention of the computer, Charney (1955) noted that the *primitive equations* would be best suited for numerical prediction of the atmosphere, and the concept of modeling was reborn.. These are called primitive equations not because they are crude or simplistic, but because they contain the fundamental physics of the atmosphere. Using the Eulerian framework in x-y-p coordinates, they can be written in the following form:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \omega\frac{\partial u}{\partial p} + 2\Omega v \sin\phi - \frac{\partial \Phi}{\partial x} + F_x \qquad (1)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \omega\frac{\partial v}{\partial p} - 2\Omega u \sin\phi - \frac{\partial \Phi}{\partial y} + F_y \qquad (2)$$

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \qquad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \qquad (4)$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + \omega\left(\frac{RT}{c_p p} - \frac{\partial T}{\partial p}\right) + \frac{\dot{Q}}{c_p} \qquad (5)$$

$$\frac{\partial q}{\partial t} = -u\frac{\partial q}{\partial x} - v\frac{\partial q}{\partial y} - \omega\frac{\partial q}{\partial p} + \dot{E} - \dot{P} \qquad (6)$$

where F_x and F_y are "friction terms" which modify the momentum equations via boundary layer turbulent drag, and by horizontal and vertical transport of momentum by turbulent eddies above the boundary layer (generally called *diffusion* in models). $\dot{Q}/c_p$ is the diabatic heating rate which can be written as:

$$\dot{Q} = \dot{Q}_L + \dot{Q}_C + \dot{Q}_R + \dot{Q}_S$$

where $\dot{Q}_L$ is "large-scale" latent heat of condensation which can be directly calculated in the model, where $\dot{Q}_C$ is "small-scale" latent heat of condensation released by clouds which cannot be directly calculated in the model. One of the most difficult problems in modeling is how to formulate expressions for the net effect of $\dot{Q}_C$ (which is a "subgrid-scale" phenomenon, meaning in this case every single cloud in the atmosphere cannot be computed in the model) in terms of the large-scale dependent variables, which is called the *parameterization problem*. $\dot{Q}_R$ is the radiative heating rate, and incorporates the affect of longwave and shortwave radiation processes. Since radiative transfer is strongly dependent on cloud and moisture distribution, $\dot{Q}_R$ is also dependent on the parameterization problem. Finally, $\dot{Q}_S$ represents sensible heat flux from the earth's surface (for example, arctic air moving over the Gulf of Mexico will warm as it acquires warmth from the water).

The precipitation rate is written as $\dot{P} = \dot{P}_L + \dot{P}_C$, and is known once $\dot{Q}_L$ and $\dot{Q}_C$ are known. $\dot{E}$ is the evaporation rate which can be due to moisture flux from the surface and/or evaporation of precipitation.

How a numerical forecast is made

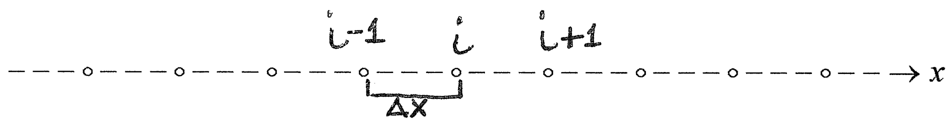
A *numerical model* is a computer program (usually FORTRAN) which makes forecasts in successive short-time increments Δt (currently about 4 min in the ETA model) out to a desired time period (48 h for the ETA model, and 10 days for the MRF). The methodology is as follows:

1) Obtain observations of the prognostic variables *u, v, T,* and *q* (usually at 00 and 12 Z to coincide with sounding data)
2) Compute ϕ from (3) and ω from (4)
3) Compute F_x , F_y, $\dot{Q}$, $\dot{E}$, and $\dot{P}$ and the other terms (advection, pressure gradient, and Coriolis terms) on the right hand sides of (1), (2), (5), and (6).
4) Iterate the four prognostic equations forward in time by Δt (using a "do loop") to obtain new values for *u, v, T,* and *q*
5) Repeat steps 2 to 4 until the forecast is completed

Since there are many ways to formulate the equations and parameterization schemes, and because many numerical procedures exist to solve these equations on a computer, no two numerical models are alike. This is why different models produce slightly different forecasts, although remarkably models generally produce similar predictions. Another difference is how spatial derivative terms (ie., $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, and $\frac{\partial}{\partial p}$) are computed. There are basically three techniques: 1) the finite difference method; 2) the spectral method; and 3) the finite element method. The spectral method, which is used for "long-range" models such as the AVN, MRF, and ECMWF models (and for climate change models as well), will be discussed in later classes. Finite elements are beyond the scope of the class, and are generally not used much in meteorology (although it is used very much in oceanography and engineering). The finite difference technique is used for "short-range" models such as the NGM and ETA, and in mesoscale models such as the MM5, RAMS, COAMPS, and GFDL models, and is described next.

The Finite difference method

Let *T(x)* be a differentiable function of *x*. In the finite difference method, we represent *T(x)* by a set of its values at discrete points in *x*, or at *grid points*. Let us identify each grid point by an integer value of *i*, as shown below. The interval in *x* between two adjacent grid points, Δx, is the *grid size*. We assume Δx is constant and *x* of the *i*-th grid point is given by $x_i = i\Delta x$.



Let us use the notation $T_{i-1} = T(x_{i-1})$, $T_i = T(x_i)$, $T_{i+1} = T(x_{i+1})$, etc. The following finite difference expressions may be used as an approximation for *DT/Dx* at the point *i*:

Forward finite difference: $$\frac{T_{i+1} - T_i}{\Delta x}$$

Backward finite difference: $$\frac{T_i - T_{i-1}}{\Delta x}$$

Centered finite difference: $$\frac{T_{i+1} - T_{i-1}}{2\Delta x}$$

In general, centered differencing is the preferred method of representing spatial derivatives.
In the ETA model, Δx =48 km, while in the "meso-ETA" Δx =29 km.

Time derivatives may be represented in a similar fashion. Let us use the notation $T^{\tau-1}$ to represent a value of *T* one time interval Δt (or *time step*) in the past. Likewise, T^{τ} represents the current value of *T*, and $T^{\tau+1}$ is a future value of *T* one time step in the future. The following expressions may be used as an approximation for *DT/Dt* at the point *i*:

Forward in time: $$\frac{T_i^{\tau+1} - T_i^{\tau}}{\Delta t}$$

Backwards in time: $$\frac{T_i^{\tau} - T_i^{\tau-1}}{\Delta t}$$

Centered in time (also called the "leapfrog" scheme): $$\frac{T_i^{\tau+1} - T_i^{\tau-1}}{2\Delta t}$$

The leapfrog scheme is the preferred scheme of the three above, although many, many other time iteration schemes exist which cannot be discussed here which are clever combinations of these three. For example, the ETA model uses a scheme called a "modified Euler scheme," and the NGM uses a scheme called the Lax-Wendroff scheme. As mentioned previously, the time step for the ETA model is 4 min.

Examples using the 1-D advection equation

Recall that the 1-D temperature advection equation is:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x}$$

The "Forward in Time, Backwards in Space" (FTBS) finite difference form is:

$$\frac{T_i^{\tau+1} - T_i^{\tau}}{\Delta t} = -u_i^{\tau}\frac{T_i^{\tau} - T_{i-1}^{\tau}}{\Delta x} \qquad (7)$$

All the variables are known except $T_i^{\tau+1}$, hence (7) may be rewritten:

$$T_i^{\tau+1} = T_i^{\tau} - \frac{u_i^{\tau}\Delta t}{\Delta x}\left(T_i^{\tau} - T_{i-1}^{\tau}\right) \quad (8)$$

A forecast of temperature change due to zonal wind advection may now be performed by plugging in the current values of u_i^{τ}, T_{i-1}^{τ}, and T_i^{τ} for the prescribed grid spacing Δx and computing $T_i^{\tau+1}$ one time step (Δt) into the future. Perform the same for $T^{\tau+1}$ at all other grid points, as well as for $u_i^{\tau+1}$. Then plug these new values into the right hand side of (8), and do another time step. This procedure is repeated until the forecast time is reached.

This, of course, looks quite straightforward, but this is where a misunderstanding of numerical solutions can be very dangerous. It turns out that FTBS schemes will not accurately forecast advective processes at all, and in fact will artificially diffuse (wash out) values with time! Even more strange, it turns out some schemes will not work at all! For example, the "Forward in time, Centered in Space" (FTCS) scheme will incorrectly produce exponentially growing solutions that will crash the computer program. *The choice of a correct finite difference scheme is crucial in model development!*

The centered in time centered in space scheme (CTCS, also called the leapfrog scheme) is one of the best time iteration schemes and computes advective processes well. For this reason, the leapfrog method will be explored in a homework exercise. However, the leapfrog scheme has its own set of errors which will be discussed in a later course.

Concluding remarks

Please realize that models are not perfect, and forecast errors result for many reasons These errors occur partially because:

1) derivatives are approximated by finite differences in time and space
2) parameterization of diabatic processes
3) inadequate observation network (especially over ocean basins)