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Anomalous Gradient Winds: Existence and Implications

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ABSTRACT—The solutions to the quadratic gradient wind equation are examined using 200-mb data for two selected cases. Under conditions of anticyclonic flow, both solutions are shown to have relevance. The normal solution is valid for wind speeds less than twice geostrophic, whereas the "anomalous" solution applies for wind speeds greater than twice the geostrophic value.

For both cases, wind speeds of more than three times the geostrophic value were detected synoptically over large areas of the United States. Areas of negative absolute vorticity were generally found within the anomalous flow regions. The absolute value of the absolute vorticity approached the local value of the Coriolis parameter.

Although cause-effect relationships are difficult to ascertain, the coexistence of negative absolute vorticity and anomalous flow implies stability. Instability would be expected in regions where the anomalous flow alone was observed. Manifestation of this instability in the form of clear-air turbulence (CAT) is suggested.

Many numerical models exclude anomalous flow and/or negative absolute vorticity by use of physical constraints. Some aspects of this exclusion are examined.

1. INTRODUCTION

Wind-pressure or wind-geopotential relationships form an integral part of numerical prediction models. When one initializes data for numerical models, such forms as the quasi-geostrophic approximation (Charney 1949) or the balanced wind (Cressman 1959) may be used. Another wind-geopotential relationship is the gradient wind, which is defined under a balance of Coriolis, pressure-gradient, and centrifugal forces for frictionless, horizontal flow. This balance has been described in several forms by Petterssen (1956), Haltiner and Martin (1957), and Hess (1959).

When expressed in natural coordinates, the tangential and centripetal components of the acceleration become, respectively,

$$\dot{V}=\frac{dV}{dt}=fV_g \sin\alpha \tag{1}$$

and

$$\frac{V_G^2}{R_T}=fV_g\cos\alpha-fV_G \tag{2}$$

where R_T, the radius of trajectory curvature, is negative for anticyclonic flow. (All symbols used in this paper are defined in table 1.) For large-scale flow patterns outside the Tropics, in which α is typically less than 30°, the value of the cosine in eq (2) will be nearly unity and can be excluded from consideration.

Solving eq (2) for $1/V_G$ and inverting, we obtain two solutions for the gradient wind:

$$V_{GN}=\frac{V_g}{\frac{1}{2}+\left(\frac{1}{4}+\frac{V_g}{fR_T}\right)^{1/2}}=\frac{V_g}{\frac{1}{2}+\left(\frac{1}{4}-\frac{V_g}{V_I}\right)^{1/2}} \tag{3a}$$

and

$$V_{GA}=\frac{V_g}{\frac{1}{2}-\left(\frac{1}{4}+\frac{V_g}{fR_T}\right)^{1/2}}=\frac{V_g}{\frac{1}{2}-\left(\frac{1}{4}-\frac{V_g}{V_I}\right)^{1/2}}. \tag{3b}$$

V_I is the inertial wind, which describes a balance between centripetal and Coriolis accelerations on a frictionless geopotential surface without a pressure gradient. The

TABLE 1.—*The symbols used in this paper*

f	Coriolis parameter
g	acceleration due to gravity
K_s	streamline curvature
K_T	trajectory curvature
R_T	radius of trajectory curvature $=1/K_T$
u	west-east wind component
v	south-north wind component
V	$(u^2+v^2)^{1/2}$
V_{act}	actual (observed) wind
V_g	geostrophic wind
V_G	gradient wind
V_{GN}	normal gradient wind
V_{GA}	anomalous gradient wind
V_I	inertial wind $=-fR_T$
z	height above sea level
α	angular deviation in a mathematical (counterclockwise) sense from the geostrophic wind
β	meteorological wind direction (positive in a clockwise sense)
ψ	stream function
θ	potential temperature (use as a subscript denotes an isentropic surface; $\theta=$constant)
ζ_a	absolute vorticity
∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

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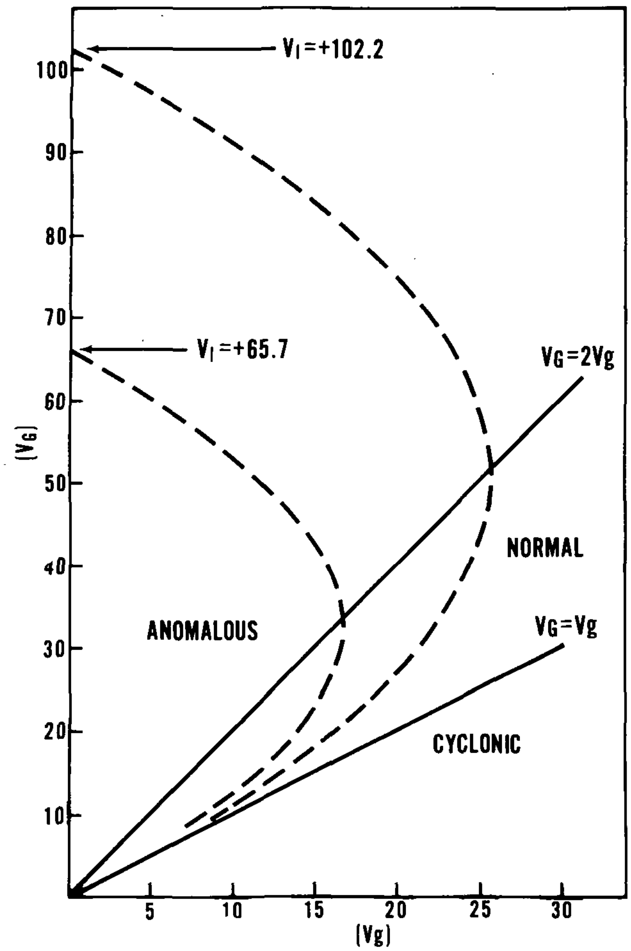

FIGURE 1.—Plot of geostrophic, V_g, versus gradient, V_G, winds (m/s) showing the double solution for the gradient wind equation during anticyclonic flow. Dashed curves represent two values of $(-fR_T)$, which becomes inertial wind when $V_g=0$.

anomalous solution to eq (2) is eq (3b). For anticyclonic values of R_T, a zero radical produces the following relationship as a lower limit for the anomalous solution:

$$V_G=2V_g. \tag{4}$$

For anomalous gradient balance, therefore, the wind speed must be greater than or equal to twice the geostrophic speed. Figure 1 shows these features graphically. Equation (3a) is satisfied in two regions of this diagram. One region is below $V_G=V_g$ for cyclonic flow, and the other is the "normal" region for normal anticyclonic flow. In contrast, eq (3b) is satisfied in the "anomalous" region above $V_G=2V_g$. Also shown are two sample curves (dashed) of a family of $(-fR_T)$ curves. At $V_g=0$, these quantitites become the indicated inertial winds. The normal and anomalous gradient winds for anticyclonic flow are easily found for a given V_g by moving vertically to the two intersections with a given $(-fR_T)$ line.

Previously, consideration of the existence of anomalous gradient flow has ranged from Brunt (1939), who discounted it as an algebraic accident, to Hess (1959), who reasoned that it would not occur frequently because of the high wind speeds required. In general, researchers have minimized the importance of anomalous flow because it requires:

1. The existence of clockwise rotation in space, a feature for which there is no known mechanism on the large scale.
2. A large or infinite energy supply due to the high wind speeds.

Examining the first of these conditions, we note that a body in solid rotation possesses an angular velocity, V/R, while the earth's horizon has an angular velocity of $f/2$. Assuming that α is small, eq (2) can be rewritten as

$$V_G\left(\frac{V_G}{R_T}+\frac{f}{2}\right)=fV_g-\frac{fV_G}{2}. \tag{5}$$

If $V_G>2V_g$, then the quantity in parentheses must be negative. For anticyclonic flow, the absolute value of V_G/R_T must exceed $f/2$. Rotation, therefore, is clockwise in space in the Northern Hemisphere.

Alaka (1961) solved eq (2) directly. He substituted $1/K_T$ for the radius of trajectory curvature and wrote for the two anticyclonic solutions:

$$V_{GN}=\frac{-f}{2\,K_T}\left[1-\left(1-\frac{4\,K_TV_g}{f}\right)^{1/2}\right] \tag{6a}$$

and

$$V_{GA}=\frac{-f}{2\,K_T}\left[1+\left(1-\frac{4\,K_TV_g}{f}\right)^{1/2}\right]. \tag{6b}$$

Binomial expansion of the radical with appropriate simplification yields

$$V_{GN}=V_g-\frac{K_TV_g^2}{f}+\ldots \tag{7a}$$

and

$$V_{GA}=\frac{-f}{K_T}-V_g+\frac{K_TV_g^2}{f}+\ldots. \tag{7b}$$

For anticyclonic flow ($K_T<0$), the normal solution of eq (7a) reduces to geostrophic flow as the radius of trajectory curvature becomes very large. The velocity given by the anomalous solution increases infinitely as K_T approaches zero.

Substitution of V_I, the inertial wind (Newton 1959, Angell 1962), into eq (7b) yields

$$V_{GA}=V_I-V_g-\frac{V_g^2}{V_I}-\ldots. \tag{8a}$$

As the geostrophic wind approaches zero, the anomalous solution approaches that for the inertial wind. This upper bound, combined with eq (4), yields limits for anomalous flow indicated by

$$V_I\geq V_{GA}\geq 2\,V_g. \tag{8b}$$

Large-scale atmospheric flow patterns that satisfy the anomalous solution can, therefore, exist without requiring an infinite energy supply.

2. EXISTENCE OF ANOMALOUS FLOW FIELDS

While Newton and Persson (1962) were unable to find any cases in which the actual wind approached twice the geostrophic value in the subtropical jet stream, other researchers have reported the occurrence of anomalous flow. We cite case studies by Gustafson (1953) of 700-mb winds under anticyclonic flow and by Angell (1960) of 300-mb balloon trajectories in a subtropical ridge as just two examples. Other researchers [e.g., Alaka (1961), Black and Anthes (1971)] have shown the existence of anomalous flow in the outflow layer of many, if not all, hurricanes.

When we examined daily weather maps for a 2-yr period, we found several situations over North America where anomalous flow may have occurred (mostly at 200 mb). Most cases appear to have a duration of a day or so. The best documented pair of cases will be discussed here. Case 1 (1200 GMT, Dec. 5, 1963) occurred in connection with a subtropical jet stream; case 2 (1200 GMT, July 19, 1965) did not. In both cases, the anomalous solution to the gradient wind equation best described the observed conditions over portions of the United States. Data sources included teletype reports and *Northern Hemispere Data Tabulations*.[3] Mexican upper air data were unavailable for case 1; aircraft data were not used for either case.

Case 1

For this case, the wind reports at Midland, Tex., were examined in great detail for temporal as well as vertical consistency (fig. 2). The observed winds are reasonable on several accounts:

1. Shears were large, but not excessive.
2. The elevation angle for the 1200 GMT observation on December 5 was always larger than 10° because of light winds below 6 km.
3. Average winds at Midland for December 1963 showed no systematic bias toward high speeds.

Figures 3 and 4 show observed winds and heights and hand-analyzed isotach and height fields at 200 mb for 1200 GMT on Dec. 5, 1963. We also prepared an isogon analysis (not shown). Gridpoint values were extracted for computation of several derived fields. The grid interval was 111 km. Note that in figures 3 and 4 we have used the Brownsville, Tex., 12,000-m wind, since this was the last wind observation in the sounding. The same level at Del Rio, Tex., had 64 m/s, while only 76 m higher, at 200 mb, the wind was 54 m/s We believe the stronger wind is correct, based on vertical consistency; this choice somewhat reduces the strength of anticyclonic shear and negative absolute vorticity south of Del Rio.

To isolate the region of anomalous flow, we evaluated the ratio V_{act}/V_g (fig. 5A). Ratio values greater than two (shaded) indicate anomalous flow; we found gridpoint values exceeding three near Midland.

[3] Available at National Climatic Center, NOAA, Asheville, N.C.

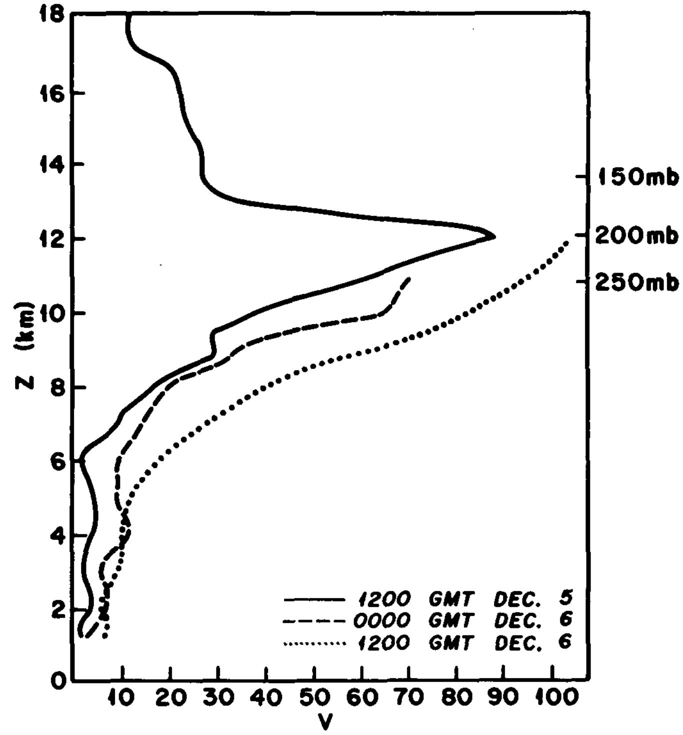

FIGURE 2.—Observed vertical profiles of wind speed (m/s) for three consecutive soundings at Midland, Tex., in December 1963.

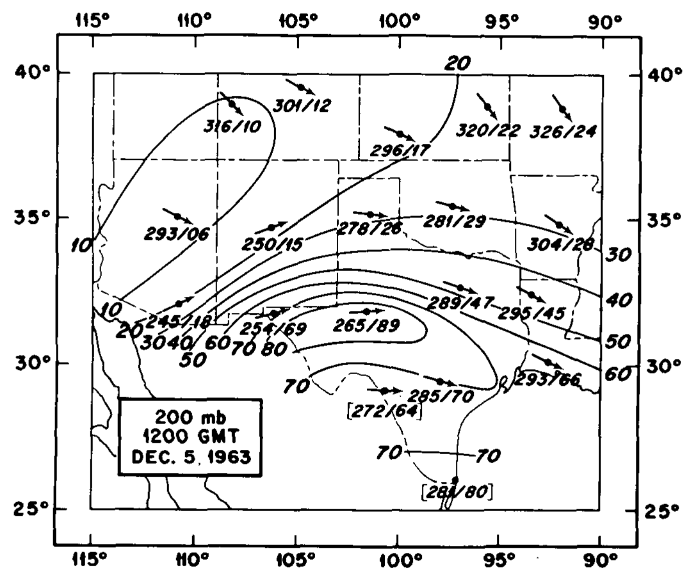

FIGURE 3.—Observed 200-mb winds (direction in degrees, speed in m/s) and hand-analyzed isotach field for 1200 GMT on Dec. 5, 1963. Brownsville, Tex., report (in brackets) was taken from 12,000-m level where the report terminated just below 200 mb. Del Rio, Tex., report (also in brackets) was also taken from 12,000 m.

Winds in excess of twice geostrophic appeared to have existed from 0000 GMT on December 5 to 0000 GMT on the 6th over southern and central Texas through a depth of only 2–3 km. The time scale and depth of the phenomena agree well with those found by Gustafson (1953) The shape of the general flow pattern remained anticyclonic and relatively unchanged through the period. After 1200

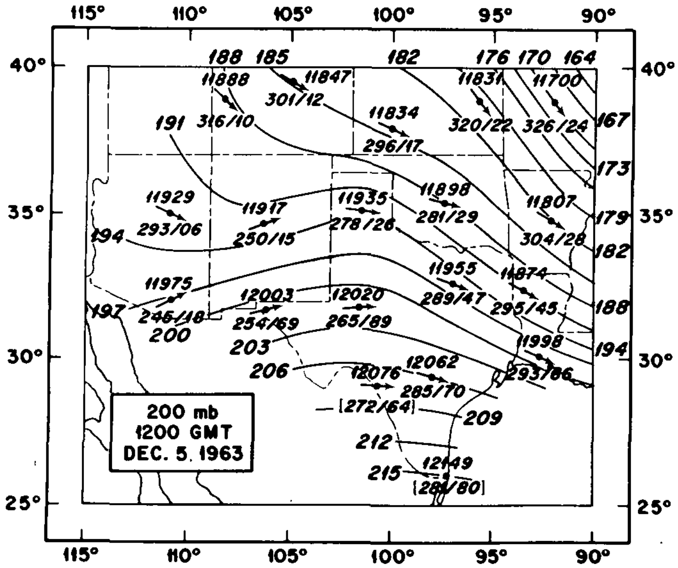

FIGURE 4.—Observed 200-mb heights (m) and winds (direction in degrees, speed in m/s) and hand-analyzed height field for 1200 GMT on Dec. 5, 1963. Brownsville and Del Rio, Tex., reports are as in figure 3.

GMT on December 5, warming below the maximum wind level occurred to the south of the jet axis; in this case, the pressure field appeared to adjust to the wind field.

Since the height analysis was used to determine the geostrophic wind, we hydrostatically checked all radiosonde reports. A few corrections of up to 25 m were made. The resulting analysis produced a smooth north–south profile of heights through Midland. A consistent and smooth height analysis indicating a much stronger geostrophic wind at Midland could be achieved only if all three Texas stations to the south of Midland reported heights too low, by 20 m or more, over several observation times.

Equations (3a) and (3b) were used in an attempt to compare the two solutions for eq (2) with the observed winds. Of the three quantities needed to evaluate the radical, the radius of trajectory curvature, R_T, proved the most difficult to obtain. Two methods were available for computing R_T. Alaka (1961) derived the relationship

$$K_T=\frac{1}{R_T}=\frac{1}{V^3}\left(\frac{udv}{dt}-\frac{vdu}{dt}\right). \tag{9}$$

However, to evaluate the total derivatives, he assumed a steady state for which the components of motion of the system were known.

A more realistic method makes use of Blaton's equation (Petterssen 1956). If we let

$$K_T=\frac{1}{V}\frac{d\beta}{dt}$$

and

$$K_s=\frac{\partial\beta}{\partial s},$$

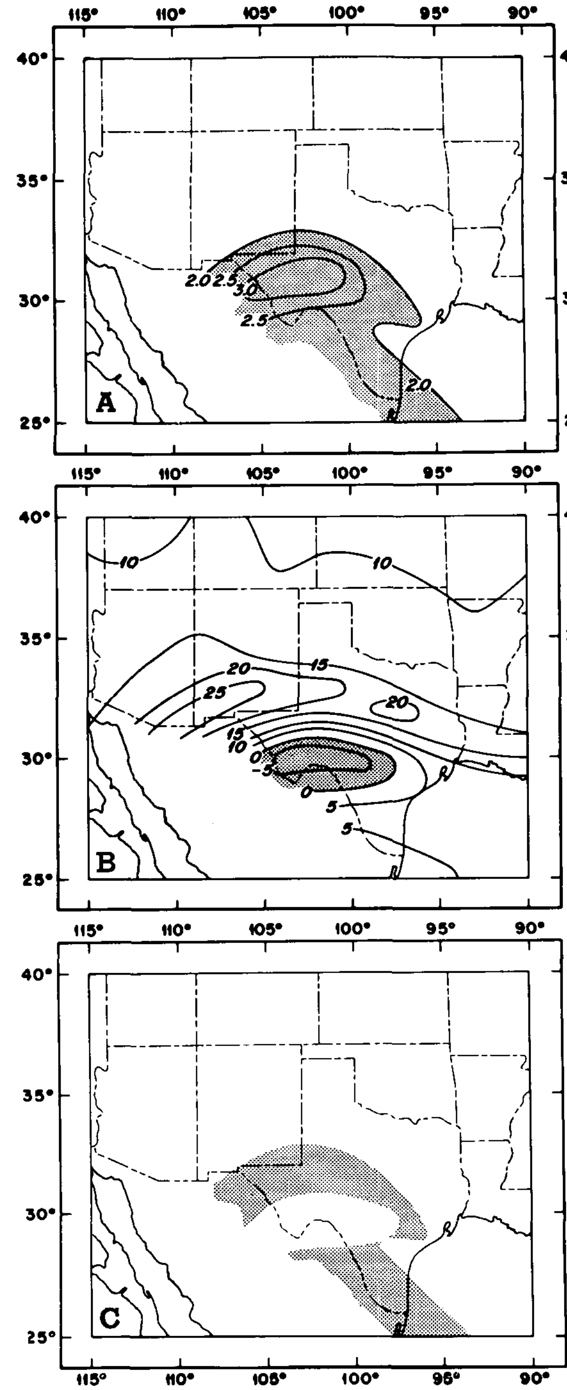

FIGURE 5.—Calculated values at 200 mb for case 1 at 1200 GMT on Dec. 5, 1963. (A) is the ratio of actual to geostrophic wind (V_{act}/V_g)—shaded areas with values larger than two indicate anomalous flow and (B) is absolute vorticity ($\times 10^{-5}s^{-1}$) from observed winds—shaded areas indicate negative absolute vorticity. In (C), the shaded areas indicate dynamically unstable regions based on eq (11); that is, either anomalous flow or negative absolute vorticity is observed.

then

$$K_T=K_s+\frac{1}{V}\frac{\partial\beta}{\partial t} \tag{10}$$

for frictionless, horizontal flow. For this equation, we measured the anticyclonic curvature along the flow through

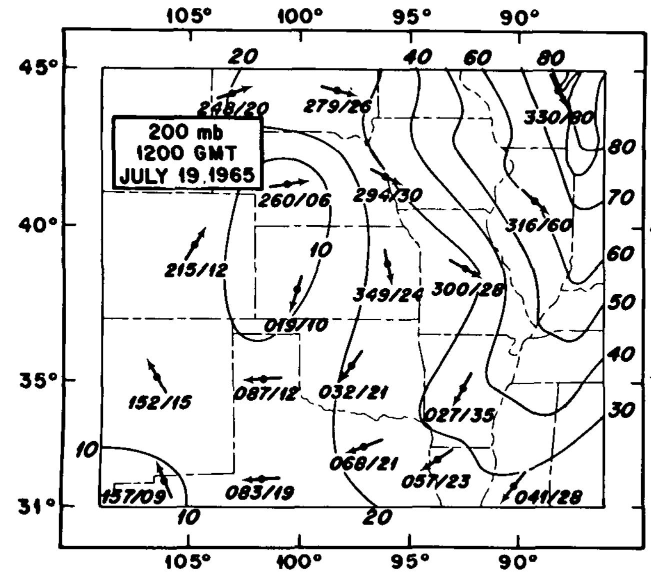

FIGURE 6.—Observed 200-mb winds (direction in degrees, speed in m/s) and hand-analyzed isotach field for 1200 GMT on July 19, 1965.

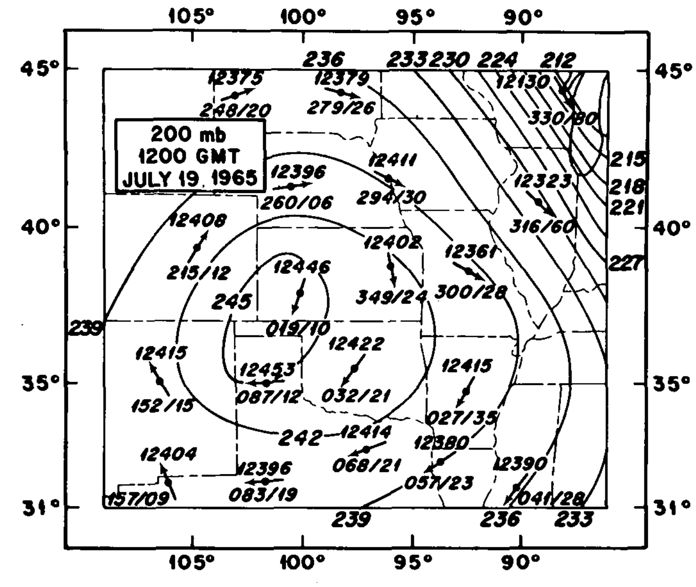

FIGURE 7.—Observed 200-mb heights (m) and winds (direction in degrees, speed in m/s) and hand-analyzed height field for 1200 GMT on July 19, 1965.

Midland over a homogeneous region representative of the speed maximum. While vertical motions, although only a few centimeters per second in magnitude, could become important in the curvature calculation if allowed to operate for relatively large time periods, we had little information on their magnitude and only some idea of their sign.

We used eq (3a), (3b), and (10) to compute the normal and anomalous solutions for a gridpoint near Midland for case 1. For $V=89$ m/s, $V_g=29$ m/s, $f=7.8 \times 10^{-5}$ s^{-1}, $K_T \approx -1700$ km, and $\alpha=0°$, we found that $V_{GA}=90$ m/s and $V_{GN}=43$ m/s. It is clear from this example that the anomalous solution is a better estimate of the observed wind.

From gridpoint data, we computed a field of absolute vorticity (fig. 5B). The area of negative absolute vorticity (shaded) south of the wind maximum is due primarily to the strong anticyclonic shears in the horizontal wind speed. The region of negative absolute vorticity is located within and covers a smaller area than the region of anomalous flow (fig. 5C). Black and Anthes (1971) found almost complete superposition of anomalous flow and negative absolute vorticity in the hurricane outflow layer.

Case 2

Lower wind speeds characterized this case, and observed height gradients, if taken literally, indicated a flow opposite to the observed flow between some stations. All radiosonde reports were hydrostatically checked, and corrections of up to 35 m were made at many stations. The observed winds and a hand-analyzed isotach analysis at 200 mb are shown in figure 6; a smooth 200-mb height field and original data for 1200 GMT on July 19, 1965, are given in figure 7. From these, we computed the ratio V_{act}/V_g, using the same methods given for case 1. Wind speeds exceeding twice geostrophic (shaded) are widespread with maximum gridpoint values of over three indicated for northern Arkansas (fig. 8A). If the gradient wind calculation at Little Rock, Ark., is made for $V_g=10$ m/s and $R_T=-570$ km for no cross-contour flow, we find that the observed wind of 35 m/s compares to the anomalous wind of 36 m/s and a normal gradient wind of 14 m/s. The pattern of anomalous flow persisted for at least 24 hr with the region contracting in size with time. In this case, decreasing velocities rather than increasing gradients were observed. This indicates that the transition from anomalous to normal balance is not a unique process. One can reason that the transition from normal to anomalous flow is probably not unique either.

The anomalous flow was not confined to a jet stream since geostrophic winds in portions of the anomalous flow region were as low as 5 m/s. The field of absolute vorticity for case 2 (fig. 8B) shows negative values (shaded) over a rather large area compared to figure 5 for case 1. As with case 1, the anomalous flow was confined to a relatively shallow layer and covered a larger area than the negative absolute vorticity (fig 8C).

3. INSTABILITY CONSIDERATIONS

Alaka (1961) assumed gradient equilibrium in a vortex in which the angular speed, ω, is a function of distance from the vortex center. Instability of motion results when

$$\left[\zeta_a\left(\frac{2V}{R_T}+f\right)\right]_\theta<0. \tag{11}$$

For both cases studied, anomalous flow occurs in regions with little temperature variation along streamlines, such that motions along an isentropic surface can be expressed

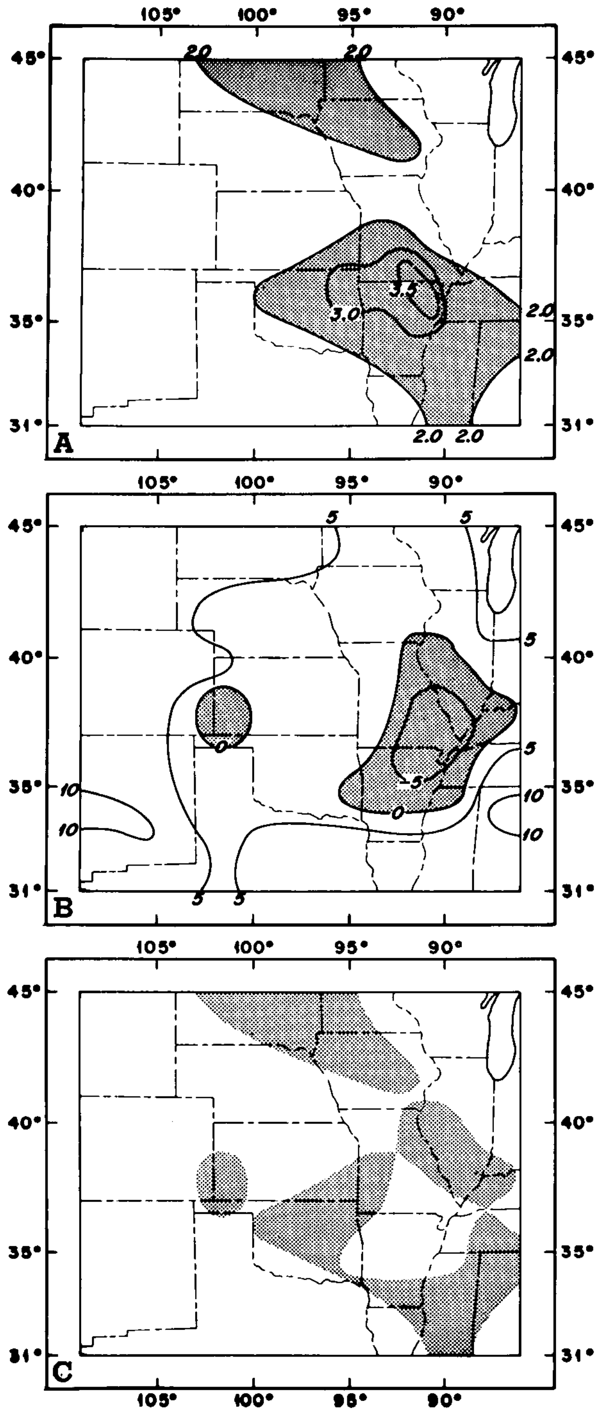

FIGURE 8.—Same as figure 5 for 1200 GMT on July 19, 1965.

nearly identically with motions on a constant-pressure surface. Instability results either when the absolute vorticity becomes negative or when $|2V_G/R_T|$ exceeds f. The latter condition is that of anomalous flow.

For both cases, the contour patterns changed little during the period of anomalous flow. The cross-contour flow was much less than 30°. While the assumptions used to derive eq (11) were not explicitly met, the instability criterion should be applicable.

The ratio (V_{act}/V_g) and absolute vorticity fields were superimposed to locate the unstable regions. These are shaded in figures 5C and 8C for cases 1 and 2, respectively. For both cases, the area of negative absolute vorticity lies primarily within the region of anomalous flow. This results in a smaller unstable region than would be observed with the anomalous flow alone.

Moore et al. (1968) independently examined case 1 for purposes of studying clear-air turbulence (CAT). Their data (0000 GMT, Dec. 6, 1963) indicated a large region of moderate to severe CAT reports over central Texas. Our analysis indicated that this region was unstable 12 hr earlier. While the contention that the smaller scale instability is an outcome of larger scale processes may prove valid, more research will be needed to establish firmly the existence of such a relationship. The CAT may be connected with the large observed shears rather than the instability implied by eq (11), which could be due either to negative absolute vorticity or anomalous flow. Unfortunately, aircraft observations of CAT frequently go unreported, or aircraft avoid regions in which CAT is suspected.

The existence of either negative absolute vorticity or anomalous flow does not imply, necessarily, the existence of the other. Angell (1962) stated that negative absolute vorticity is *usually* attained through the existence of large anticyclonic shears. The criterion for anomalous flow is not dependent upon the horizontal shear although anomalous flow, by definition, occurs only under anticyclonic curvature. In the cases examined here, the coexistence of negative absolute vorticity and anomalous flow seems likely particularly in the regions of highest velocity, where the strongest shears are observed.

Sorenson (1964) noted an increase in the number of CAT reports as the anticyclonic curvature of wind fields increased under the influence of a thermal ridge. He suggests that the centrifugal force term, mV^2/R, may operate to release eddy energy into the system whenever the curvature reaches a critical value. Under circumstances when R can become quite large, this critical value is, in our estimation, the value of R needed to produce anomalous flow. While no sharp thermal ridge was observed in the cases studied for this report, the fact that a critical value of the curvature may have been reached (or exceeded) under the existing flow pattern may be used to explain the occurrence of CAT over part of the region of anomalous flow.

4. NUMERICAL PREDICTION ASPECTS

Wind speeds satisfying the anomalous solution [eq (3b)] can differ markedly from geostrophic or normal gradient winds (as shown here). Failure to incorporate the anomalous wind phenomenon into the initialization of wind and geopotential data, coupled with the inability of most, if not all, numerical models to forecast the phenomenon, can bias numerical products for which anomalous flow is either observed initially or for which conditions would support its generation. Further, if there is indeed a coexistence of anomalous flow and negative absolute vorticity, then the exclusion of anomalous flow will exclude the negative absolute vorticity through reduction in shear values. Conversely, exclusion of negative absolute vorticity will not necessarily exclude anomalous flow, at least for the cases examined here.

The existence of large regions of negative absolute vorticity would pose serious problems to any model that utilizes the balance equation,

$$f\nabla^2\psi-\left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial \psi}{\partial y}\right)+2\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}-\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right]=g\nabla^2 z. \quad (12)$$

Houghton (1968) derived an ellipticity criterion, eq (13), for eq (12). If

$$g\nabla^2 z+\frac{f^2}{2}-\left(\frac{\psi}{a}\right)\left(\frac{\partial f}{\partial y}\right)>0 \quad (13)$$

at every point within a region, then eq (12) could be solved as a boundary value problem. However, if the region becomes hyperbolic, then a different solution method must be used. Shuman (1957) derived a similar argument in which the ellipticity is guaranteed if the absolute vorticity remains nonnegative. 'Arnason (1958), likewise, developed a criterion of positive absolute vorticity but remarked that this alone might not be a sufficient criterion.

Usually, the heights are smoothed slightly to produce an elliptic field everywhere. 'Arnason (1958) stated that the scale of pressure systems dealt with in numerical weather prediction is such that a slight smoothing of the analysis makes eq (12) elliptic over the entire map.

For case 2, we found from inspection of real-time forecasts that the then current (1963) three-level baroclinic model was unable to predict the occurrence of high wind speeds in the region of weak pressure gradients through the Middle Mississippi Valley. It was in this region that the observed winds were greater than three times the geostrophic winds at some gridpoints.

It is possible that a smoothing process within the model was the cause of the large errors. However, it would appear initially that the difficulty lies in the physics of the model. The model was simply incapable of predicting this type of flow field.

At present, the National Meteorological Center (NMC) is using a more sophisticated, six-level, primitive-equation model to produce wind and geopotential forecasts for constant-pressure surfaces over much of the Northern Hemisphere. This model has a grid length of 381 km. During the initialization stage when heights and winds are being matched, it is likely that difficulties occur when a balance equation is used for reasons already mentioned. Subsequent forecasts based on these initial data could fail to allow either type of dynamic instability due to the large grid scale (over three times as large as we used here), the smoothing-desmoothing scheme (U.S. Department of Commerce 1968), or the averaging in the finite-differencing scheme. All of these factors may reduce negative absolute vorticity values, if not eliminate them entirely. At times, the 200-mb forecasts show marked signs of synoptic scale instability in which a wind maximum curls anticyclonically across height contours toward higher heights. Stackpole (1969) stated that on occasion the isotachs under such conditions on the 24- or 30-hr 200-mb forecast appear somewhat unusual. While he believed this to be related to excessive negative absolute vorticity, he could not apply any cause–effect relationship to the problem.

In August 1971, NMC began to use objectively analyzed wind fields rather than winds obtained from the nonlinear balance equation (U.S. Department of Commerce 1971). This change was expected to produce better forecasts particularly *south* of *strong* jets (i.e., on the anticyclonic shear side).

5. CONCLUSIONS

We found that:

1. Large atmospheric regions exist that appear to satisfy more closely the anomalous gradient balance than either normal gradient or geostrophic balances.
2. Negative absolute vorticity exists on a synoptic scale in midlatitudes.
3. An apparent relationship exists between CAT occurrences and synoptically unstable regions.

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REFERENCES

Alaka, M. A., "The Occurrence of Anomalous Winds and Their Significance," *Monthly Weather Review*, Vol. 89, No. 11, Nov. **1961**, pp. 482–494.

Angell, J. K., "An Analysis of Operational 300-mb Transosonde Flights from Japan in 1957–58," *Journal of Meteorology*, Vol. 17, No. 1, Feb. **1960**, pp. 20–35.

Angell, J. K., "The Influence of Inertial Instability Upon Transosonde Trajectories and Some Forecast Implications," *Monthly Weather Review*, Vol. 90, No. 6, June **1962**, pp. 245–251.

'Arnason, Geirmundur, "A Convergent Method for Solving the Balance Equation," *Journal of Meteorology*, Vol. 15, No. 2, Apr. **1958**, pp. 220–225.

Black, Peter G., and Anthes, Richard A., "On the Asymmetric Structure of the Tropical Cyclone Outflow Layer," *Journal of the Atmospheric Sciences*, Vol. 28, No. 8, Nov. **1971**, pp. 1348–1366.

Brunt, David, *Physical and Dynamical Meteorology*, 2d Edition, Cambridge University Press, London, England, **1939**, 428 pp.

Charney, Jule G., "On a Physical Basis for Numerical Prediction of Large-Scale Motions in the Atmosphere," *Journal of Meteorology*, Vol. 6, No. 6, Dec. **1949**, pp. 371–385.

Cressman, George P., "An Operational Objective Analysis System," *Monthly Weather Review*, Vol. 87, No. 10, Oct. **1959**, pp. 367–374.

Gustafson, Arthur F., "On Anomalous Winds in the Free Atmosphere," *Bulletin of the American Meteorological Society*, Vol. 34, No. 5, May **1953**, pp. 196–201.

Haltiner, George J., and Martin, Frank L., *Dynamical and Physical Meteorology*, McGraw-Hill Book Co., Inc., New York, N.Y., **1957**, 470 pp.

Hess, Seymour L., *Introduction to Theoretical Meteorology*. Henry Holt and Co., New York, N.Y., **1959**, 362 pp.

Houghton, David D., "Derivation of the Elliptic Condition for the Balance Equation in Spherical Coordinates," *Journal of the Atmospheric Sciences*, Vol. 25, No. 5, Sept. **1968**, pp. 927–928.

Moore, Richard L., Wasko, Peter E., and Lem, Paul A., "Diagnostic Synoptic Case Studies of Clear Air Turbulence," *Proceedings of the Third National Conference on Aerospace Meteorology, New Orleans, Louisiana, May 6–9, 1968*, American Meteorological Society, Boston, Mass., **1968**, pp. 528–538.

Newton, Chester W., "Axial Velocity Streaks in the Jet Stream: Ageostrophic 'Inertial' Oscillations," *Journal of Meteorology*, Vol. 16, No. 6, Dec. **1959**, pp. 638–645.

Newton, Chester W., and Persson, Anders V., "Structural Characteristics of the Subtropical Jet Stream and Certain Lower-Stratospheric Wind Systems," *Tellus*, Vol. 14, No. 2, Stockholm, Sweden, May **1962**, pp. 221–241.

Petterssen, Sverre, *Weather Analysis and Forecasting*, 2d Edition, Volume 1, McGraw-Hill Book Co., Inc., New York, N.Y., **1956**, 428 pp.

Shuman, Frederick G., "Numerical Methods in Weather Prediction: I. The Balance Equation," *Monthly Weather Review*, Vol. 85, No. 10, Oct. **1957**, pp. 329–332.

Sorenson, John E., "Synoptic Patterns for Clear Air Turbulence," *UAL Meteorology Circular* No. 56, United Air Lines, Chicago, Ill., Dec. 21, **1964**, 64 pp.

Stackpole, John D., Development Division, National Meteorological Center, Suitland, Md., Apr. 10, **1969** (personal communication).

U.S. Department of Commerce, "Divergent Initialization of the 6-Layer (PE) Numerical Prediction Model," *Technical Procedures Bulletin* No. 13, Weather Bureau, ESSA, Suitland, Md., May 13, **1968**, 4 pp.

U.S. Department of Commerce, "New Initialization Procedure for the 6-Layer (PE) Numerical Prediction Model," *Technical Procedures Bulletin* No. 65, National Weather Service, NOAA, Silver Spring, Md., Aug. 12, **1971**, 5 pp.

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