
6.3 Numerical Integration: Newton–Cotes Formulas

Definite integrals are encountered in a wide range of applications, generally in the form

$$\int_a^b f(x) dx$$

where $f(x)$ is the integrand and a and b are the limits of integration. The value of this definite integral is the area of the region between the graph of $f(x)$ and

the x -axis, bounded by the lines $x = a$ and $x = b$. As an example of a definite integral, consider the relation between the bending moment M and shear force V along the longitudinal axis x of a beam, defined by

$$M_2 - M_1 = \int_{x_1}^{x_2} V dx$$

where M_2 is the bending moment at position x_2 and M_1 is the bending moment at x_1 . In this case, the integrand is shear force $V(x)$ and the limits of integration are x_1 and x_2 .

The integrand may be given analytically or as a set of discrete points. Numerical integration is used when the integrand is given as a set of data or, the integrand is an analytical function, but the antiderivative is not easily found. To carry out numerical integration, discrete values of the integrand are needed. This means that even if the integrand is an analytical function, it must be discretized and the discrete values will be used in the calculations.

6.3.1 Newton–Cotes Formulas

Newton–Cotes formulas provide the most commonly used integration techniques and are divided into two categories: closed form and open form. In closed form schemes, the data points at the endpoints of the interval are used in calculations; the trapezoidal and Simpson's rules are closed Newton–Cotes formulas. In open form methods, limits of integration extend beyond the range of the discrete data; the rectangular rule and the Gaussian quadrature (Section 6.4) are open Newton–Cotes formulas.

The main idea behind Newton–Cotes formulas is to replace the complicated integrand or data with an easy-to-integrate function, usually a polynomial. If the integrand is an analytical function, it is first discretized, and then the polynomial that interpolates this set is found and integrated. If the integrand is a set of data, the interpolating polynomial is found and integrated.