Name______________________

Exercise 1: Power series

10 pts each

Binomial expansion

1) Expand $(2x+3y)^4$ (see http://tutorial.math.lamar.edu/Classes/CalcII/BinomialSeries.aspx). Use binomial expansion as defined below.

If n is any positive integer then,

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

$$= a^n + na^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 + \cdots + nab^{n-1} + b^n$$

where,

$$\binom{n}{i} = \frac{n(n-1)(n-2)\cdots(n-i+1)}{i!} \qquad i = 1, 2, 3, \ldots n$$

$$\binom{n}{0} = 1$$

2) Write the first three terms in the binomial series for $(8-4x)^{1/3}$. See example 2 in http://tutorial.math.lamar.edu/Classes/CalcII/BinomialSeries.aspx .

3) A common approximation for very small x using the negative binomial expansion is

$$(1-x)^{-1} = 1 + x + x^2 + H.O.T \approx 1 + x\ .$$

Show the negative binomial expansion steps below out to order 2

Plug in x=0.1 and check the validity of this approximation

4) Read http://www.animations.physics.unsw.edu.au/jw/doppler.htm#general . The Doppler frequency shift for both a moving observer and a moving source is:

$$f_{shift} = f\frac{v + v_o}{v - v_s}$$

Using the identity in 3, show the steps to get $f_{shift} \approx f(1 + v_o/v)(1 + v_s/v)$ for small v_o and small v_s

Expand this term further and neglect products and small numbers to get a final expression.

Taylor series expansion

You can use http://www.derivative-calculator.net/ to assist with derivatives, but show all steps.

5) Write the Taylor series expansion for $f(x) = x^{1/3}$ at x_o out to the third derivative.

6) Compute $(9.1)^{1/3}$ where $x_o = 9$ using this expansion. Compare it to the true solution.

7) Let $x_o = 0$ and write out a Taylor series expansion of e^x out to the nth derivative. (Note: when $x_o = 0$ for a Taylor series, it's also known as a Maclaurin series).

8) For small *x*, what is an approximation for e^x? This is used for many physics and meteorology math approximations.

9) Use a Taylor Series expansion out to the second derivative for $(1-x)^{-1}$ about $x_o = 0$. Note you will get the same answer as question 3.

10) For $f(x) = \ln(x)$ about x_o one solution is $\ln(x_o) + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k{x_o}^k}(x - x_o)^k$. See example 7 in http://tutorial.math.lamar.edu/Classes/CalcII/TaylorSeries.aspx . Solve this on a spreadsheet for x=2.1 and $x_o = 2$. Show me the spreadsheet.

a. Summing 4 rows for k= 1 to 3
b. Using an array formulation for k=1 to 100

BONUS QUESTIONS

11) Solve the following.

For $f(x) = e^{-6x}$ about $x_o = -4$, the solution is $e^{24} + \sum_{k=1}^{\infty} \frac{e^{24}(-6)^k}{k!}(x+4)^k$. See http://tutorial.math.lamar.edu/Problems/CalcII/TaylorSeries.aspx for details.
Solve this out to k=100 on a spreadsheet for x=-4.1.

The series $\pi = \sum_{k=1}^{\infty} \frac{16(-1)^{k+1}}{(2k-1)5^{2k-1}} - \sum_{k=1}^{\infty} \frac{4(-1)^{k+1}}{(2k-1)239^{2k-1}}$ proposed by Machin in 1706 converges quickly. Solve this accurate to at least 15 digits on a spreadsheet using this series.

For $f(x) = \ln(x)$ about x_o the solution can also be written as $x_o + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(\frac{x}{e^{x_o}} - 1\right)^k$.
Solve this on a spreadsheet for x=2.1 and $x_o = 2$. Show me the spreadsheet.

a. Summing 10 rows for k= 1 to 9
b. Using an array formulation for k=1 to 100