6.2.4 Derivative Estimates for Nonevenly Spaced Data

The finite-difference formulas to approximate derivatives of various orders require that the data be equally spaced. Also, Richardson's extrapolation is applicable only to evenly spaced data and it computes better estimates by sequentially reducing the spacing by half. These techniques are appropriate if the data is equally spaced or if the data is generated by uniform discretization of a known function, such as that in Examples 6.1 and 6.2.

Empirical data—such as data resulting from experimental measurements—on the other hand, are often not evenly spaced. For these situations, one possible way to approximate the derivative is as follows: (1) consider a set of three consecutive data points that contains the point at which the derivative is to be estimated, (2) fit a second-degree Lagrange interpolating polynomial (Section 5.5) to the set, and (3) differentiate the polynomial and evaluate at the point of interest. The derivative estimate obtained in this manner has the same accuracy as that offered by the central difference formula, and exactly matches it for the case of equally spaced data.

TABLE 6.4

Data in Example 6.4

x_i	y_i
0	1
0.3	0.8228
0.8	0.4670
1.1	0.2617
1.3	0.1396

Example 6.4: Nonevenly Spaced Data

For the data in Table 6.4, approximate the first derivative at $x = 0.9$ using the data at 0.3, 0.8, and 1.1.

SOLUTION

The data is not evenly spaced. We will consider the set of three consecutive points 0.3, 0.8, and 1.1, which includes the point of interest $x = 0.9$, and fit a second-degree Lagrange interpolating polynomial to the set. Letting $x_1 = 0.3$, $x_2 = 0.8$, and $x_3 = 1.1$, we find

$$p_2(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}(0.8228) + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}(0.4670)$$

$$+ \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}(0.2617)$$

$$= 0.0341x^2 - 0.7491x + 1.0445$$

Differentiation yields $p_2'(x) = 0.0682x - 0.7491$ so that $p_2'(0.9) = -0.6877$.