6.2.5 MATLAB® Built-In Functions `diff` and `polyder`

The MATLAB® built-in function `diff` can be used to estimate derivatives for both cases of equally spaced and not equally spaced data. A brief description of `diff` is given as

```
diff(X) calculates differences between adjacent elements of X.

If X is a vector, then diff(X) returns a vector, one element
shorter than X, of differences between adjacent elements:

            [X(2)-X(1) X(3)-X(2)... X(n)-X(n-1)]

diff(X,n) applies diff recursively n times, resulting in the
nth difference. Thus, diff(X,2) is the same as diff(diff(X)).
```

Equally spaced data: Consider a set of equally spaced data $(x_1, y_1), \ldots, (x_n, y_n)$, where $x_{i+1} - x_i = h$ $(i = 1, \ldots, n-1)$. Then, by the description of `diff`, the command `diff(y)./h` returns the $(n-1)$-dimensional vector

$$\begin{bmatrix} \dfrac{y_2 - y_1}{h} & \cdots & \dfrac{y_n - y_{n-1}}{h} \end{bmatrix}$$

The first component is the first-derivative estimate at x_1 using the forward-difference formula; see Equation 6.4. Similarly, the second component is the derivative estimate at x_2. The last entry is the derivative estimate at x_{n-1}. As an example, consider $f(x) = e^{-x}\sin(x/2)$, $x = 1.2, 1.4, 1.6, 1.8$, of Example 6.1. We find an estimate for $f'(1.4)$ as follows:

```
>> h=0.2;
>> x=1.2:h:1.8;
>> y=[0.1701 0.1589 0.1448 0.1295];
% Values of f at the discrete x values
>> y_prime=diff(y)./h

y_prime =

  -0.0560    -0.0702    -0.0767
```

Since 1.4 is the second point in the data, it is labeled x_2. This means an estimate for $f'(1.4)$ is provided by the second component of the output `y_prime`. That is, $f'(1.4) \cong -0.0702$. This agrees with the earlier numerical results in Table 6.1.

Nonequally spaced data: Consider a set of nonevenly spaced data $(x_1, y_1), \ldots, (x_n, y_n)$. Then, by the description of `diff`, the command `diff(y)./diff(x)` returns the $(n-1)$-dimensional vector

$$\begin{bmatrix} \dfrac{y_2 - y_1}{x_2 - x_1} & \cdots & \dfrac{y_n - y_{n-1}}{x_n - x_{n-1}} \end{bmatrix}$$

The first component is the first-derivative estimate at x_1 using the forward-difference formula, the second one is the derivative estimate at x_2, while the last entry is the derivative estimate at x_{n-1}.

As mentioned in the description of `diff` above, `diff(y,2)` is the same as `diff(diff(y))`. So, if $y = [y_1 \ \ldots \ y_n]$, then `diff(y)` returns

$$\begin{bmatrix} y_2 - y_1 & y_3 - y_2 & \cdots & y_n - y_{n-1} \end{bmatrix}_{(n-1)\text{ dim}}$$

and `diff(y,2)` returns

$$[(y_3 - y_2) - (y_2 - y_1) \quad (y_4 - y_3) - (y_3 - y_2) \quad \ldots \quad (y_n - y_{n-1}) - (y_{n-1} - y_{n-2})]_{(n-2)\text{ dim}}$$

which simplifies to

$$\begin{bmatrix} y_3 - 2y_2 + y_1 & y_4 - 2y_3 + y_2 & \cdots & y_n - 2y_{n-1} + y_{n-2} \end{bmatrix}$$

The first component is the numerator in the three-point forward difference formula for estimating the second derivative at x_1; see Equation 6.9. Similarly, the remaining components agree with the numerator of Equation 6.9 at $x_2, \ldots, x_{n-2}$. Therefore, for an equally spaced data $(x_1,y_1), \ldots, (x_n,y_n)$, an estimate of the second derivative at $x_1, x_2, \ldots, x_{n-2}$ is provided by

```
diff(y,2)./h^2
```

The MATLAB built-in function `polyder` finds the derivative of a polynomial:

```
polyder Differentiate polynomial.

  polyder(P) returns the derivative of the polynomial whose
  coefficients are the elements of vector P.

  polyder(A,B) returns the derivative of polynomial A*B.

  [Q,D] = polyder(B,A) returns the derivative of the polynomial
  ratio B/A, represented as Q/D.
```

For example, the derivative of a polynomial such as $2x^3 - x + 3$ is calculated as follows:

```
>> P = [2 0 -1 3];

>> polyder(P)

ans =

     6     0    -1
```

The output corresponds to $6x^2 - 1$.