<u>Proof that pressure is force per unit area</u>

Weight (*W*gt) is mass (*m*) times gravity (*g*), and mass is density (ρ) times volume (*Vol*):

$Wgt = mg$ and $m = \rho Vol$

Volume is an increment of Δx, Δy, and Δz, and area A is $A = \Delta x \Delta y$

$Vol = \Delta x \Delta y \Delta z = \Delta A \Delta z$ (1)

Hence

$Wgt = \rho \Delta A \Delta z g$ and if one takes the limit approaching zero $dWgt = \rho g dA dz$ (2)

Integrate the hydrostatic equation $\frac{dp}{dz} = -\rho g$ from a level *z* to the top of the atmosphere z_{top}:

$p(z) = \int_0^{z_{top}} \rho g dz$ (3)

Suppose an increment of pressure is weight per unit area integrated in an incremental cube of air.

$p(z) = \int \frac{dWht}{dA}$ (4)

But $\frac{dWht}{dA} = \rho g dz$, substitute into Eq. (4)

$p(z) = \int_0^{z_{top}} \rho g dz$ (5)

Equation 3 and 5 are the same! Pressure is the weight per unit area.