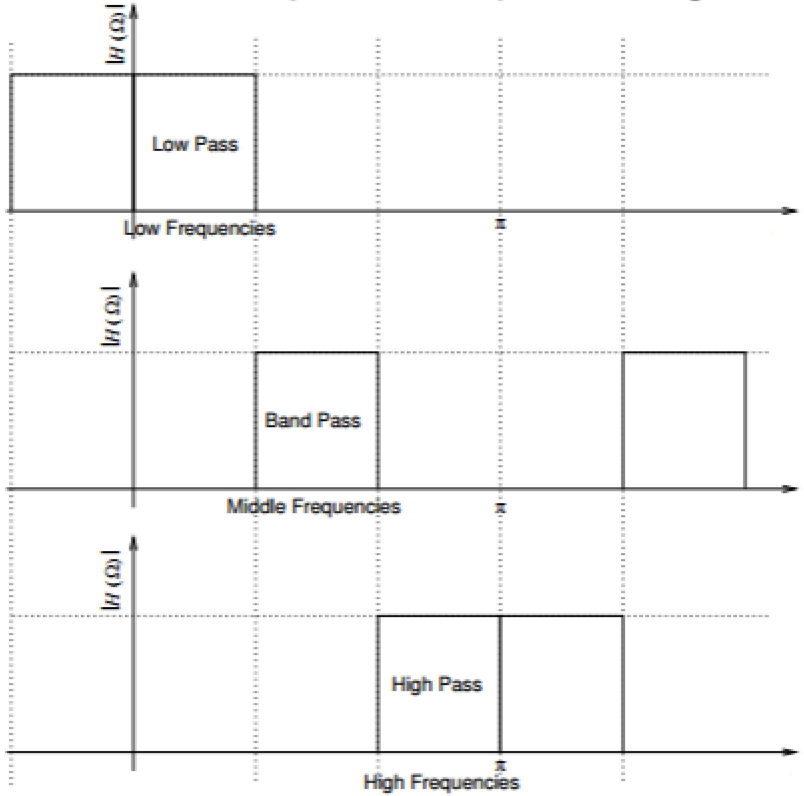
What is a Digital Filter?

Any digital system can be described as a digital filter. The word "*filter*" means to remove a part of a signal and allow another part to pass through.
The verb "*to filter*" is used in many areas of English language.
Examples

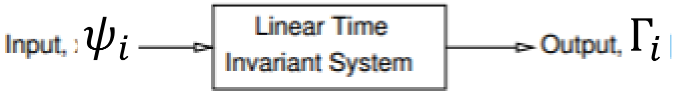
- The water filter cleans the water for drinking.
- The cook filtered the bad fruit from the good for cooking later.
- The air conditioning filters dust from the air.

What is a Digital Filter?

Often used to remove some frequencies from a signal $X($ and to allow other frequencies to pass through to the output Y).



Non-recursive digital filters



What is a *non-recursive digital filter*?

- "*Recursive*" comes from the word "to recur"
  Meaning: to repeat

A recursive filter uses past output values ($i.e.$, Γ_{i-1} for the current output Γ_i :

- *Recursive Filter Example*

$$\Gamma_i = 0.5\,\Gamma_{i-1} + 0.5\psi_i$$

A non-recursive filter only uses input values $i.e, \psi_i$,

- *Non-recursive Filter Example*

$$\Gamma_i = 0.5\psi_{i-1} + 0.5\psi_i$$

Filters are often run multiple times for the same dataset. In the nonrecursive cases, all Γ values are moved to the right side of the equation, get relabeled as Ψ, and the process is repeated. Each of these sequences is known as a "pass."

A non-recursive filter generally has the form

$$\Gamma_i = w_i \psi_i + \sum_{m=1}^{n} w_{i+m} \psi_{i+m} + \sum_{m=1}^{n} w_{i-m} \psi_{i-m}$$

where the summation of all w_i usually equals one.

If the weight is largest for i then reduces away from i, this arrangement is usually tailored to remove small wavelengths. There is some smoothing effect for each pass, but it generally retains the contributions from larger wavelengths. It’s known as a *lowpass filter*.

If the weights are equal (known as a *moving average* or *running mean*), this is a averaging effect and smooths the result, but impacts all wavelengths.

These are also called *Finite Impulse Response (FIR) filters*, as a non-recursive filter generally has a finite response to a limited “impulse” of input.

A recursive filter generally has the form

$$\Gamma_i = \alpha\Gamma_{i-1} + g(\psi_i)$$

where $|\alpha| \leq 1$. These are also called an *Infinite Impulse Response (IIR) filter*

For a recursive lowpass filter, $g(\psi_i) = (1-\alpha)\psi_i$, hence

$$\Gamma_i = \alpha\Gamma_{i-1} + (1-\alpha)\psi_i$$

where $0 \leq \alpha < 1$.

For a recursive highpass filter:

$$\Gamma_i = \alpha\Gamma_{i-1} + (1+\alpha)\psi_{i-1}$$

where $-1 \leq \alpha < 0$.

Higher order recursive filters are possible, such as a second-order version

$$\Gamma_i = \alpha_1 \Gamma_{i-1} + \alpha_2 \Gamma_{i-2} + g(\psi_i)$$

or this combination, which is two first-order filters in succession

$$\Gamma_i = (\alpha + \beta)\Gamma_{i-1} + \alpha\beta\Gamma_{i-2} + g(\psi_i)$$

It’s obvious higher orders and combination are possible as well.

A good reference is:

Otnes, R. K., and L. Enochson, 1972: Digital time series analysis. John Wiley & Sons, New York, NY, 465 pp.

Another type of recursive filter is known as the exponential smoother

$$\Gamma_i = \alpha\psi_i + (1-\alpha)\,\Gamma_{i-1}$$

where $0 \leq \alpha < 1$.

See http://en.wikipedia.org/wiki/Exponential_smoothing for more information on its properties.

Also see http://en.wikipedia.org/wiki/Moving_average for information on the weights.