

TABLE 6.3

Summary of Finite Difference Formulas for First, Second, Third, and Fourth Derivatives

Difference Formula	First Derivative	Truncation Error
2-Point backward	$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$	$O(h)$
2-Point forward	$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$	$O(h)$
2-Point central	$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2h}$	$O(h^2)$
3-Point backward	$f'(x_i) = \frac{f(x_{i-2}) - 4f(x_{i-1}) + 3f(x_i)}{2h}$	$O(h^2)$
3-Point forward	$f'(x_i) = \frac{-3f(x_i) + 4f(x_{i+1}) - f(x_{i+2}))}{2h}$	$O(h^2)$
4-Point central	$f'(x_i) = \frac{f(x_{i-2}) - 8f(x_{i-1}) + 8f(x_{i+1}) - f(x_{i+2}))}{12h}$	$O(h^4)$
Difference Formula	Second Derivative	Truncation Error
3-Point backward	$f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2}$	$O(h)$
3-Point forward	$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$	$O(h)$

3-Point central	$f''(x_i) = \frac{f(x_{i-1}) - 2f(x_i) + f(x_{i+1}))}{h^2}$	$O(h^2)$
4-Point backward	$f''(x_i) = \frac{-f(x_{i-3}) + 4f(x_{i-2}) - 5f(x_{i-1}) + 2f(x_i)}{h^2}$	$O(h^2)$
4-Point forward	$f''(x_i) = \frac{2f(x_i) - 5f(x_{i+1}) + 4f(x_{i+2}) - f(x_{i+3}))}{h^2}$	$O(h^2)$
5-Point central	$f''(x_i) = \frac{-f(x_{i-2}) + 16f(x_{i-1}) - 30f(x_i) + 16f(x_{i+1}) - f(x_{i+2}))}{12h^2}$	$O(h^4)$

Difference Formula	Third Derivative	Truncation Error
4-Point backward	$f'''(x_i) = \frac{-f(x_{i-3}) + 3f(x_{i-2}) - 3f(x_{i-1}) + f(x_i)}{h^3}$	$O(h)$
4-Point forward	$f'''(x_i) = \frac{-f(x_i) + 3f(x_{i+1}) - 3f(x_{i+2}) + f(x_{i+3}))}{h^3}$	$O(h)$
4-Point central	$f'''(x_i) = \frac{-f(x_{i-2}) + 2f(x_{i-1}) - 2f(x_{i+1}) + f(x_{i+2}))}{2h^3}$	$O(h^2)$
5-Point backward	$f'''(x_i) = \frac{3f(x_{i-4}) - 14f(x_{i-3}) + 24f(x_{i-2}) - 18f(x_{i-1}) + 5f(x_i)}{2h^3}$	$O(h^2)$
5-Point forward	$f'''(x_i) = \frac{-5f(x_i) + 18f(x_{i+1}) - 24f(x_{i+2}) + 14f(x_{i+3}) - 3f(x_{i+4}))}{2h^3}$	$O(h^2)$
6-Point central	$f'''(x_i) = \frac{f(x_{i-3}) - 8f(x_{i-2}) + 13f(x_{i-1}) - 13f(x_{i+1}) + 8f(x_{i+2}) - f(x_{i+3}))}{8h^3}$	$O(h^4)$

(continued)

TABLE 6.3 (continued)

Summary of Finite Difference Formulas for First, Second, Third, and Fourth Derivatives

Difference Formula	Fourth Derivative	Truncation Error
5-Point backward	$f^{(4)}(x_i) = \frac{f(x_{i-4}) - 4f(x_{i-3}) + 6f(x_{i-2}) - 4f(x_{i-1}) + f(x_i)}{h^4}$	$O(h)$
5-Point forward	$f^{(4)}(x_i) = \frac{f(x_i) - 4f(x_{i+1}) + 6f(x_{i+2}) - 4f(x_{i+3}) + f(x_{i+4})}{h^4}$	$O(h)$
5-Point central	$f^{(4)}(x_i) = \frac{f(x_{i-2}) - 4f(x_{i-1}) + 6f(x_i) - 4f(x_{i+1}) + f(x_{i+2})}{h^4}$	$O(h^2)$
6-Point backward	$f^{(4)}(x_i) = \frac{-2f(x_{i-5}) + 11f(x_{i-4}) - 24f(x_{i-3}) + 26f(x_{i-2}) - 14f(x_{i-1}) + 3f(x_i)}{h^4}$	$O(h^2)$
6-Point forward	$f^{(4)}(x_i) = \frac{3f(x_i) - 14f(x_{i+1}) + 26f(x_{i+2}) - 24f(x_{i+3}) + 11f(x_{i+4}) - 2f(x_{i+5})}{h^4}$	$O(h^2)$
7-Point central	$f^{(4)}(x_i) = \frac{f(x_{i-3}) + 12f(x_{i-2}) - 39f(x_{i-1}) + 56f(x_i) + 39f(x_{i+1}) + 12f(x_{i+2}) - f(x_{i+3})}{6h^4}$	$O(h^4)$