

**Table 3.2.1.** Time schemes for initial value problems  $dU/dt = F(U)$  (schemes (a)–(i));  $dU/dt = F_1(U) + F_2(U)$  (schemes (j)–(k))

(a)	$\frac{U^{n+1} - U^{n-1}}{2\Delta t} = F(U^n)$	Leapfrog (good for hyperbolic equations, unstable for parabolic equations)
(a')	$\frac{U^{n+1} - \bar{U}^{n-1}}{2\Delta t} = F(U^n);$ $\bar{U}^n = U^n + \alpha(U^{n+1} - 2U^n + \bar{U}^{n-1})$	Leapfrog smoothed with the Robert–Asselin time filter; $\alpha \sim 1\%$
(b)	$\frac{U^{n+1} - U^n}{\Delta t} = F(U^n)$	Euler (forward, good for diffusive terms, unstable for hyperbolic equations)
(c)	$\frac{U^{n+1} - U^n}{\Delta t} = F\left(\frac{U^n + U^{n+1}}{2}\right)$	Crank–Nicholson or centered implicit
(c')	$\frac{U^{n+1} - U^n}{\Delta t} = F\left(\frac{\beta U^n + (1 - \beta)U^{n+1}}{2}\right); \beta < 0.5$	Implicit, slightly damping
(d)	$\frac{U^{n+1} - U^n}{\Delta t} = F(U^{n+1})$	Fully implicit or backward
(e)	$\frac{U^* - U^n}{\Delta t} = F(U^n); \frac{U^{n+1} - U^n}{\Delta t} = F(U^*)$	Euler-backward or Matsuno: good for damping high frequency waves
(f)	$\frac{U^* - U^n}{\Delta t} = F(U^n);$ $\frac{U^{n+1} - U^n}{\Delta t} = F\left(\frac{U^n + U^*}{2}\right)$	Another predictor–corrector scheme (Heun)
(g)	$\frac{U^{n+1} - U^n}{\Delta t} = F\left(\frac{3}{2}U^n - \frac{1}{2}U^{n-1}\right)$	Adams–Bashford (second order in time).
(h)	$\frac{U^{n+1/2^*} - U^n}{\Delta t/2} = F(U^n);$ $\frac{U^{n+1/2^{**}} - U^n}{\Delta t/2} = F(U^{n+1/2^*});$ $\frac{U^{n+1^*} - U^n}{\Delta t} = F(U^{n+1/2^{**}}) \frac{U^{n+1} - U^n}{\Delta t}$ $= \frac{1}{6}[F(U^n) + 2F(U^{n+1/2^*})$ $+ 2F(U^{n+1/2^{**}}) + F(U^{n+1^*})]$	Runge–Kutta (fourth order)