Temperature advection example calculation

Overview

The 1D temperature advection is:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x}$$

which if aligned along the wind direction could be crudely approximated as:

$$\frac{T_{fcst} - T_{current}}{\Delta t} = \left|\vec{V}_{current}\right|\left(\frac{T_{upwind} - T_{downwind}}{d}\right)$$

where there are two temperature observations upwind and downwind with a distance d, and $\left|\vec{V}_{current}\right|$ is the current wind averaged between the two. $T_{current}$ is the current temperature observation located between the two points. T_{fcst} is the future temperature at this central point as the wind advects temperature for a time period Δt. This equation can be rearranged as:

$$T_{fcst} = T_{current} + \Delta t\left|\vec{V}_{current}\right|\left(\frac{T_{upwind} - T_{downwind}}{d}\right)$$

Example

Suppose the temperature in Jackson, MS, is 21°C, in Memphis, TN is 15°C, and halfway between its 18°C in Grenada, MS. The average wind between the two cities is 20 knots from the north. The distance between Jackson and Memphis is 317548 meters. If the only process affecting temperature is advection, and we assume the wind stays constant, what will the temperature in Grenada be in 2 hours?

Perform unit conversions.

$$\left|\vec{V}_{\text{current}}\right| = 20\text{ kts}\left(\frac{0.51\text{ ms}^{-1}}{1\text{ kt}}\right) = 10.2\text{ ms}^{-1}$$

$$\Delta t = 2\text{ hr}\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 7200\text{ s}$$

Compute future temperature from advection

$$T_{fcst} = 18°\text{C} + (7200\text{ s})(10.2\text{ ms}^{-1})\left(\frac{15°\text{C} - 21°\text{C}}{317548\text{ m}}\right)$$

$$T_{fcst} = 16.6°\text{C}$$

This value intuitively seems correct, since this is cold air advection, and the temperature will be falling (ignoring diabatic process). Furthermore, a 15°C air parcel would take approximately 4.324 hours to reach Grenada from Memphis (prove this to yourself). Hence, a decrease to 16.6°C in two hours makes sense.