Thermal Wind Concept

The dominant influence in the atmosphere is the differential heating that produces the warm tropics and cold polar regions, and a consequent North-south variation in the mass field. This affects the horizontal and vertical variation of the geostrophic wind field.

From the hydrostatic equation $\frac{\partial p}{\partial z} = -\rho g$, we know that upon substitution of $p = \rho RT$:

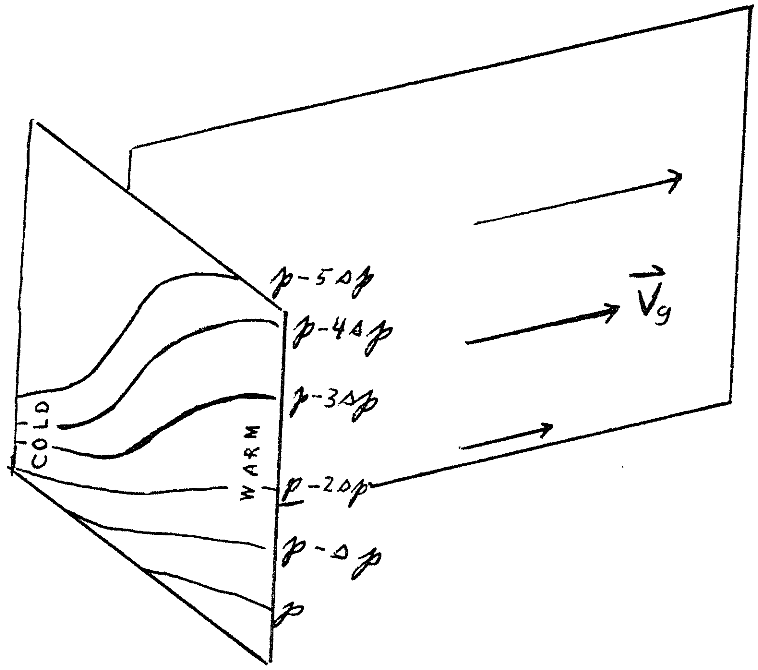
$$\frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT}$$

which shows that pressure will decrease more rapidly with height in the cold region than in the warm region.

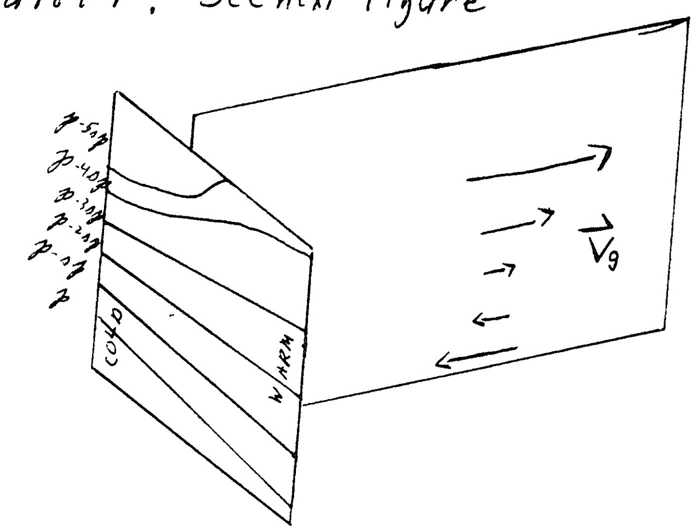
This also means the north-south pressure gradient will increase with height! It follows that westerly geostrophic winds will increase in speed with increasing height.

See Figure on next page. Note that the slope of the north-south isobars increases with height, and therefore $\vec{V}_g$ must increase as well.

As will be seen, these ideas may be incorporated into a concept called the "thermal wind."



Notice that if the temperature contrast is strong enough, even easterly surface geostrophic winds would be turned around to become strong westerlies aloft. See next figure



The main concept of this section and its implication for the atmosphere is summarized in the phrase: "The westerlies increase with height because it's colder toward the poles." Westerly winds peak at the tropopause, then $\frac{\partial T}{\partial y}$ reverses in the stratosphere (with warmer air in the poles) and the westerlies weaken above the tropopause.

It also follows that the westerlies will be enhanced in regions of localized strong temperature gradients, such as in a frontal zone. The result is that westerlies reach a maximum speed at the tropopause above the polar front, thus creating the polar-front jet stream,

See Figure:

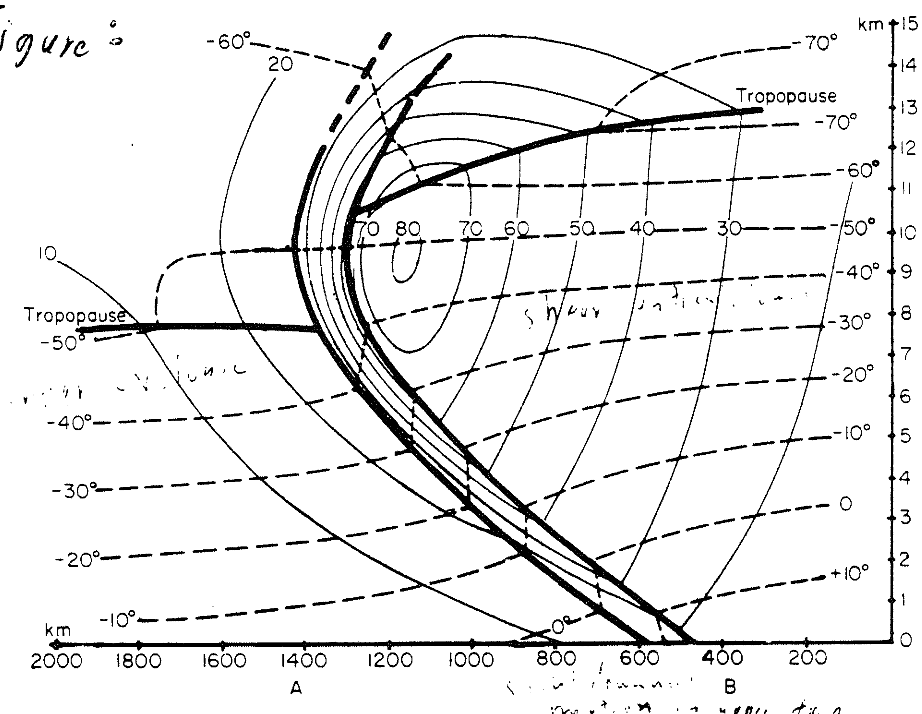


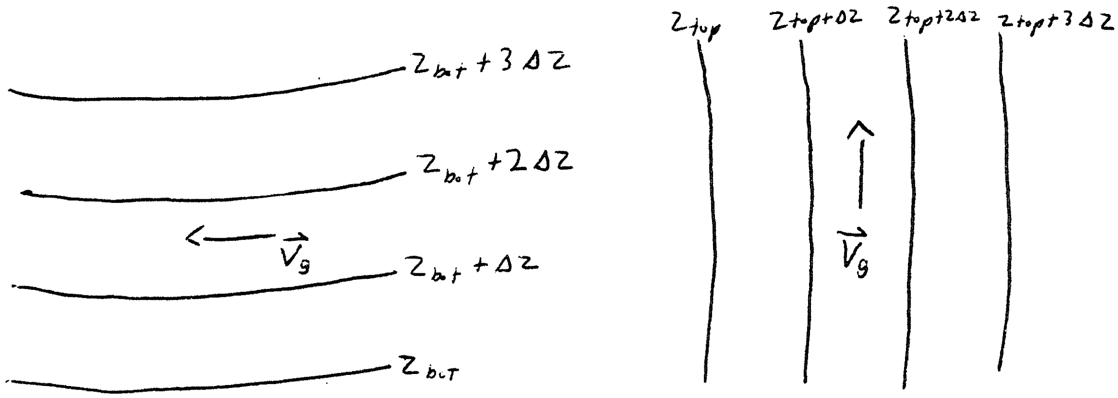
FIG. 7.4 Schematic isotherms (dashed lines, °C) and isotachs (thin solid lines, meters per second) in the polar front zone. Heavy lines are tropopauses and boundaries of frontal layer. (Adapted from analysis model by Berggren, 1952.)

It is a myth that the jet stream creates temperature contrasts at the surface as some TV weathermen state! In fact, it's the opposite! The jet stream exists due to strong temperature gradients. However, the jet stream can affect the development of weather systems — a discussion we'll delay for the synoptic classes.

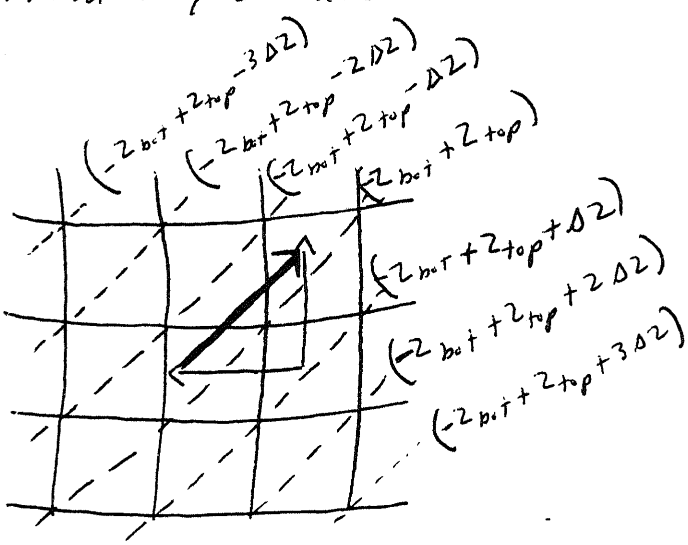
Graphical Subtraction

Before discussing the thermal wind, the method of graphical subtraction needs to be introduced.

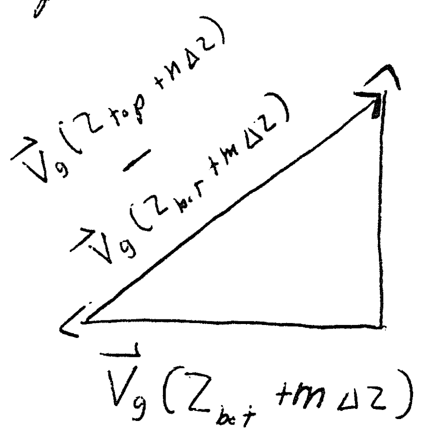
Suppose we have two pressure fields (say, 1000 and 500 mb) with height fields oriented as



By subtracting the top field from the bottom field, a thickness field may be drawn in increments of ΔZ



The graphical subtraction of $\vec{V}_g$ is



Note that the resultant vector is parallel to the the thickness lines, with the smaller ΔZ values to its left. Since ΔZ is proportional to the mean temperature, this implies the colder air is to the left of the arrow, warmer air to its right.

The resultant arrow, representing the vectorial difference of $\vec{V}_g$ at two different levels, is the <u>thermal</u> <u>wind</u>.

Here's another example, with real numbers:

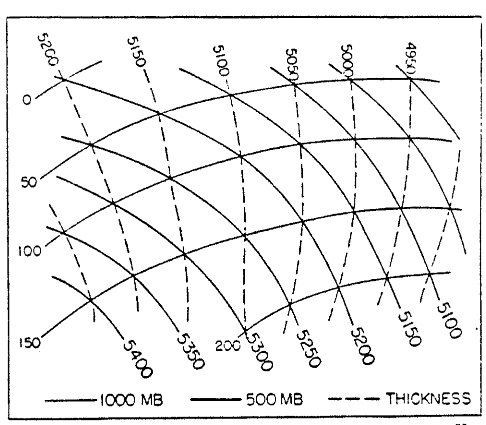
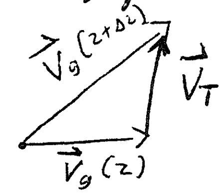


FIG. 6.2.1. Thin lines: The contours of the lower pressure surface. Heavy lines: The contours of the upper pressure surface. Broken lines: The thickness of the layer. The gradient of the thickness pattern indicates the variation in the horizontal pressure force from the lower to the upper level.

Definition of thermal wind

The classical definition of the "thermal wind" is: the vectorial difference of $\vec{V}_g$ in a height increment Δz. The notation for the "thermal wind" is usually $\vec{V}_T$, given mathematically by

or graphically



(1) $$\boxed{\vec{V}_T = \vec{V}_g\,(\underbrace{Z+\Delta Z}_{\text{thickness}}) - \vec{V}_g\,(Z)}$$

Recall that, ~~the~~ $\vec{V}_g$ may be written in vector form:

(2) $$\vec{V}_g = \frac{g}{f}\,\hat{k} \times \nabla Z$$

Therefore, by plugging (2) into (1), and using vector subtraction:

$$
\begin{aligned}
V_g\,(z+\Delta z) &= \frac{g}{f}\,\hat{k} \times \nabla(Z+\Delta Z) \\
-\quad V_g\,(z) &= \frac{g}{f}\,\hat{k} \times \nabla Z
\end{aligned}
$$

(3) $$\boxed{\vec{V}_T = \frac{g}{f}\,\hat{k} \times \nabla(\Delta Z)}$$

Furthermore, since $\Delta Z = \frac{R}{g}\,\overline{T} \ln \frac{p_{bot}}{p_{top}}$, (3) becomes

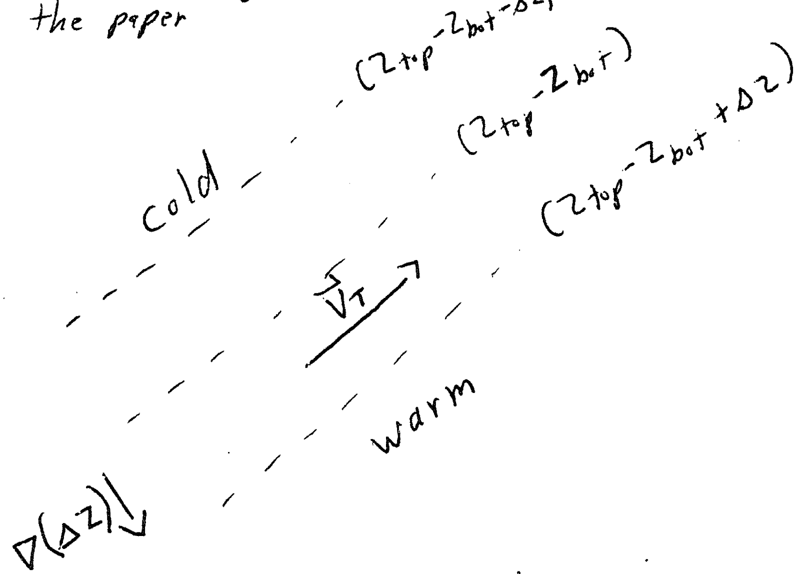
(4) $$\boxed{\vec{V}_T = \frac{R}{f} \ln \frac{p_{bot}}{p_{top}}\,\hat{k} \times \nabla \overline{T}}$$

Interpretation of thermal wind $\vec{V}_T$

Meteorologists talk about the thermal wind as though it were an actual wind. However, it is a fictitious wind, with dimensions of velocity. Why would meteorologists create a "fake wind?"

The thermal wind is a useful diagnostic tool. It can be used to analyze horizontal temperature gradients, or wind vectors at different levels. It can also be used to assess temperature advection in a layer. We will now discuss these ideas.

Using the cross-product rule, since $\frac{g}{f}\hat{k}$ points out of the paper



Eq 3 says the thermal wind "blows" parallel to thickness lines, with smaller thickness values to its left. In other words, cooler air exists on the left side of the thermal wind, and warmer air to its right. (Eq 4).

$\vec{V}_T$ is proportional to the gradient of thickness, and inversely proportional to f. The "thermal wind" is analogous to the geostrophic wind:

$\vec{V}_g$: 1) blows parallel to Z
2) smaller values of Z on its left side
3) proportional to ∇Z
4) inversely proportional to f

$\vec{V}_T$: 1) "blows" parallel to ΔZ and $\overline{T}$
2) smaller values of ΔZ on its leftside (and thus colder air)
3) proportional to $\nabla(\Delta Z)$ and $\nabla \overline{T}$
4) inversely proportional to f

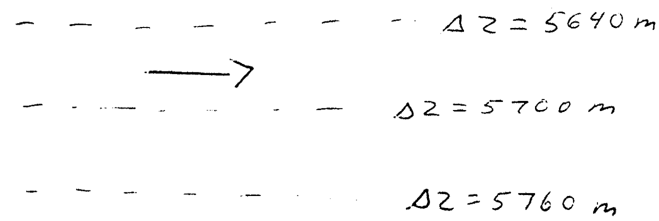
One use of $\vec{V}_T$ is that if $\vec{V}_g$ is known at one level, and $\overline{T}$ can be estimated, then $\vec{V}_g$ can be estimated at any level. For example, if $\vec{V}_g$ at 850 mb is known, and $\nabla \overline{T}$ is known in the 850-500 mb layer, then $\vec{V}_g$ at 500 mb can be computed. Another use is to make sure wind and temperature fields are consistent with one another.

Some $\vec{V}_T$ jargon

Meteorologists sometimes use thermal wind jargon in their discussions. Here's some translation:

1) "The thermal wind is strong" — there is a strong gradient of ΔZ

2) "The thermal wind is westerly" — thickness lines are oriented east-west, with smaller thickness values to the north

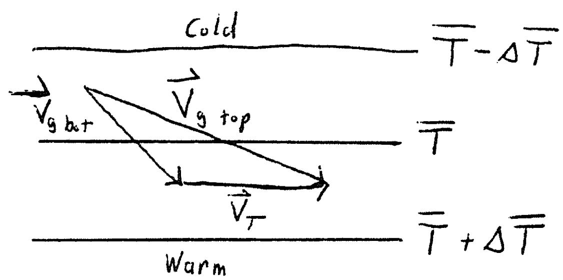


3) "The thermal wind is increasing" — $\nabla(\Delta z)$ is increasing with time

Temperature advection

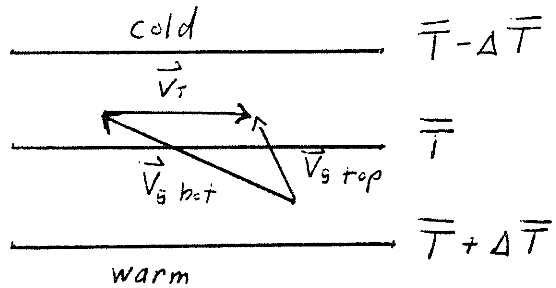
The best use of $\vec{V}_T$, however, is that the mean horizontal temperature advection in a layer can be determined.

Consider the case where the wind turns counterclockwise with height. In such a case, wind is "backing" with height.



In this case, since $\vec{V}_{g\ bot}$ and $\vec{V}_{g\ top}$ are pointing from a cold layer to warm layer, when the thermal wind indicates backing, cold air advection is occurring.

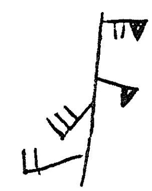
Likewise, when the wind turns clockwise with height (also called "veering"),



warm air advection is occurring.

Therefore, we can state:

1) When the wind is backing with height, cold air advection is occurring in that layer.



2) When the wind is veering with height, warm air advection is occurring in that layer.

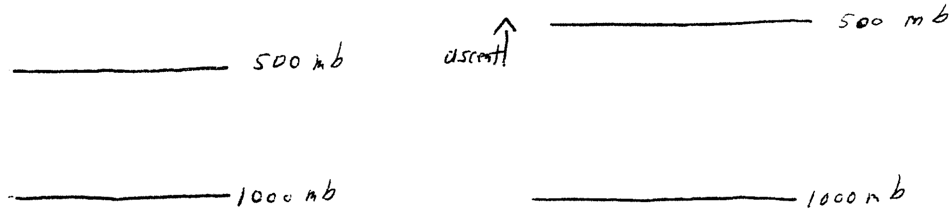
3) When the wind direction is constant with height, no temperature advection is occurring in that layer.

This is a powerful tool! It is possible to estimate horizontal temperature advection solely from the vertical profile of the wind by a single sounding (or, in some places, wind profilers).

Based on these facts

1) Future temperature changes may be inferred

2) Since warm air advection will increase the thickness of a layer, this is associated with ascending motion. For example



before warm advection — warm air advection

and vice versa for cold air advection (giving descent)

3) Changes in atmospheric stability may be inferred. For example:

Wind backing, cold air advection	top layer
Wind veering, warm air advection	bottom layer

Atmosphere becoming more unstable with time

Wind veering warm air advection	top layer
Wind backing cold air advection	bottom layer

Atmosphere becoming more stable with time

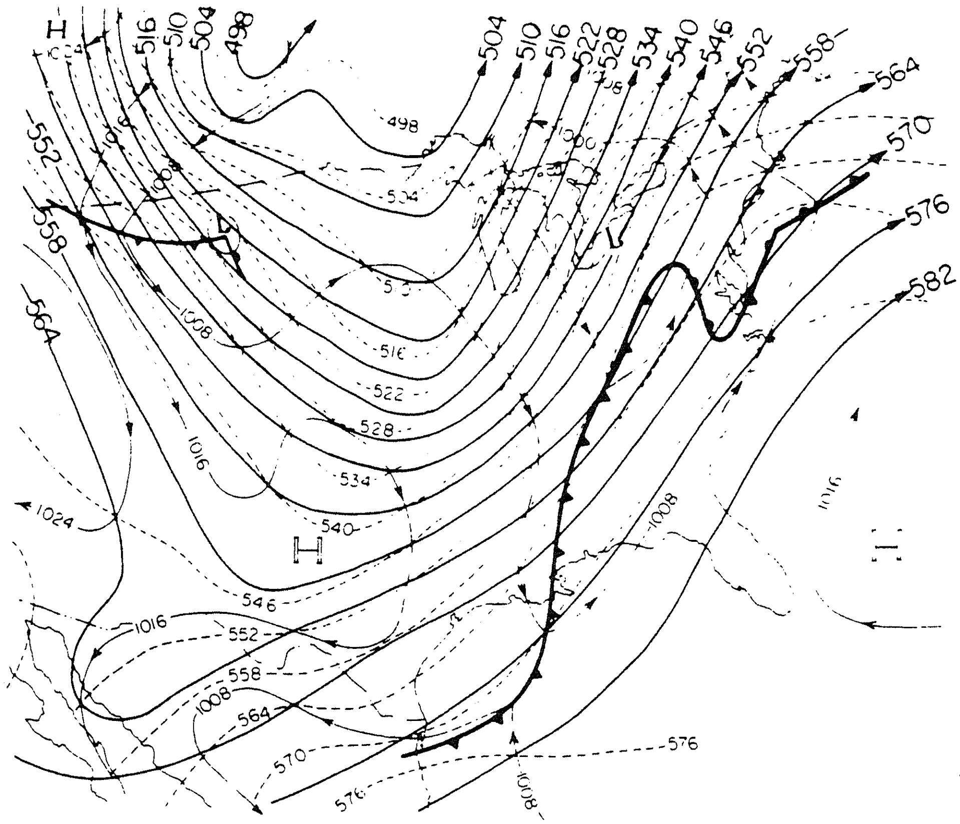


Fig. 3.21 Distribution of sea level pressure (— —), 500-mb height (——), and 1000–500-mb thickness (- - -) at 00 GCT 20 November 1964. Thickness and height contours are labeled in tens of meters. Arrows on contours denote the direction of the geostrophic wind. Letters H and L refer to maxima and minima the sea level pressure field.

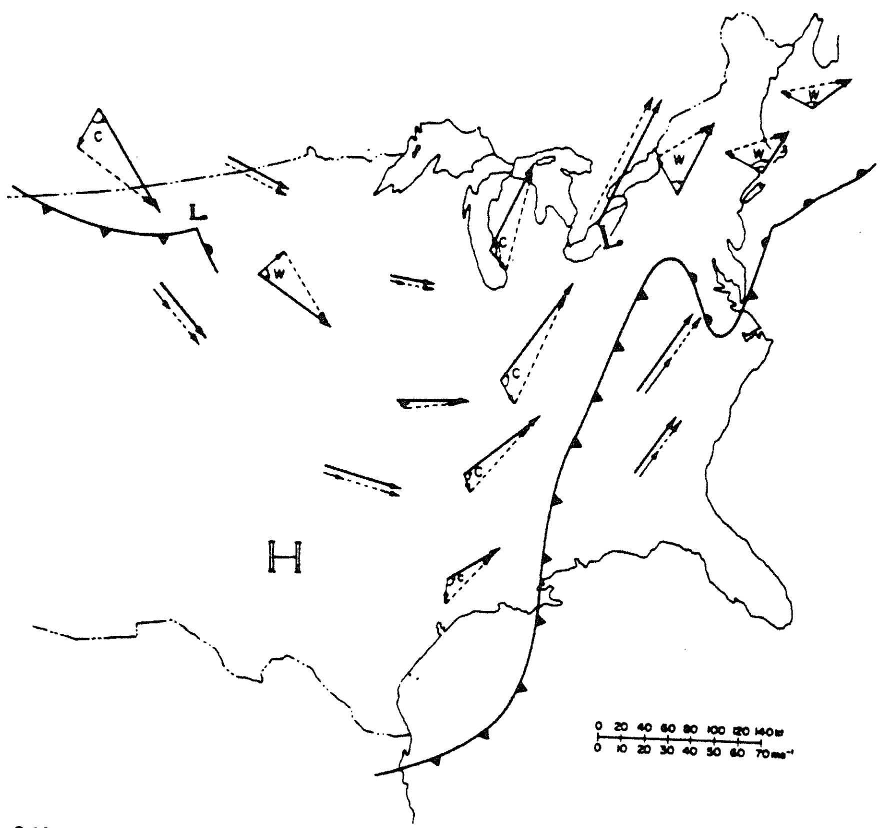
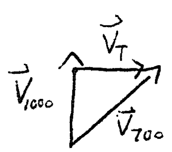


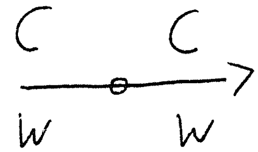
Fig. 8.18 Vertical shear of the geostrophic wind in the 1000–500-mb layer at selected locations at 00 GCT 20 November 1964. Light solid arrows represent the geostrophic wind at the 1000-mb level, heavy solid arrows represent the geostrophic wind at the 500-mb level, and dashed arrows represent the thermal wind vector for the 1000–500-mb layer. The letter C denotes cold advection and W denotes warm advection. These vectors were derived from the three sets of contours in Fig. 3.21.

4) Since warm air advection in lowest layer creates vertical motion, and because it destabilizes the atmosphere, a necessary (but not sufficient) criterion for severe weather is for the wind to veer with height from the surface to upper troposphere

5) Plots of $\vec{V}_T$ can also tell you ~~the s stability~~ the stability ~~is changing~~ NE, NW, SE, and SW of your location. Suppose for a bottom layer, $\vec{V}_T$ looks like:

veering

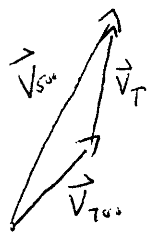


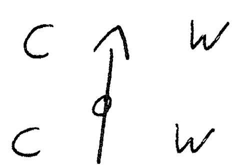


o = "you are here"

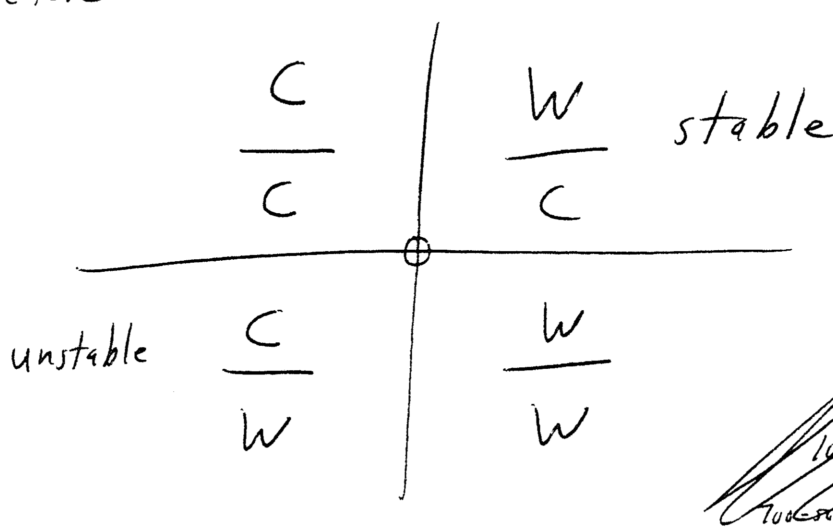
while in top layer $\vec{V}_T$ is

backing





therefore



and since you have veering 1000-700 mb, backing 700-500 mb, instability is increasing.
∴ If rain is occurring in SW, rain will probably occur locally soon.

~~and since you have veering from 1000-700 mb, and backing in 700-500, instability is in[creasing] probably ... from SW.~~

Another (erroneous) thermal wind definition

Recall

$$\vec{V}_g = \frac{g}{f}\hat{k} \times \nabla z$$

Take $\frac{\partial}{\partial p}$ of $\vec{V}_g$ gives

5 $$\frac{\partial \vec{V}_g}{\partial p} = \frac{g}{f}\hat{k} \times \nabla \frac{\partial z}{\partial p}$$

Substitution of the hydrostatic relationship

$$\frac{\partial z}{\partial p} = -\frac{1}{\rho g} = -\frac{RT}{gp}$$

into 5 gives

6 $$\boxed{\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{fp}\hat{k} \times \nabla T}$$

6 is sometimes called the thermal wind equation. However, this is not correct. The correct definition of $\vec{V}_T$ is the vector difference between $\vec{V}_g$ at two levels.

6 is a relationship describing the vertical wind shear. The shear of $\vec{V}_g$ is proportional to the horizontal temperature gradient. $\vec{V}_g$ will increase with height at a given level in a direction parallel to the

isotherms, with cold air on the left side of the shear vector. ⑥ explains the existence of westerlies aloft and jet streams in the vicinity of the polar front

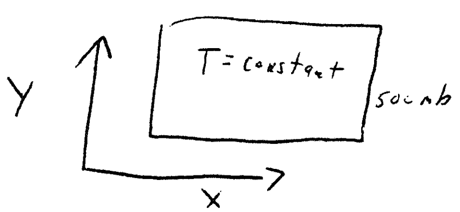
As a linear approximation:

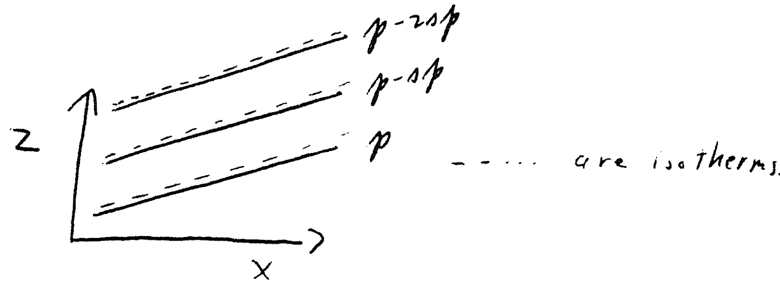
$$\vec{V}_g(z+\Delta z) = \vec{V}_g(z) + \underbrace{\underbrace{\frac{\partial \vec{V}_g}{\partial z}}_{\text{"shear" thermal wind}} \Delta z}_{\text{"vector difference" thermal wind}} = V_g(z) + \underbrace{\vec{V}_T}_{\text{"vector difference" thermal wind}}$$

Barotropic and baroclinic atmosphere

Barotropic: when density depends only on pressure. Since $p = \rho RT$, this also means temperature depends on pressure. In other words $\rho = f(p)$. Conversely $T = f(p)$

In a barotropic atmosphere, there is no temperature gradient on a constant pressure surface. In other words, temperature is constant on an isobaric surface.





Since $\vec{V}_T = \frac{R}{f} \ln \frac{p_{bot}}{p_{top}} \hat{k} \times \nabla \overline{T}$, and $\nabla \overline{T} = 0$ in a barotropic atmosphere, $\vec{V}_T = 0$!

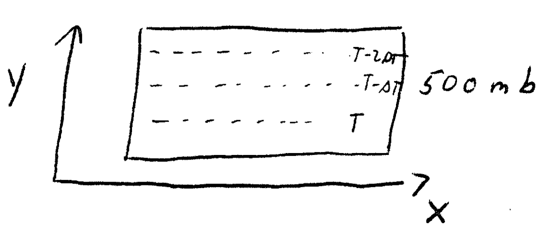
In a barotropic atmosphere, $\vec{V}_g$ is constant in speed and direction with height. This also implies no temperature advecture is occurring (of course, if T = constant on isobaric surfaces, how could advection occur!). Eq ⑥ also implies $\frac{\partial \vec{V}_g}{\partial p} = 0$.

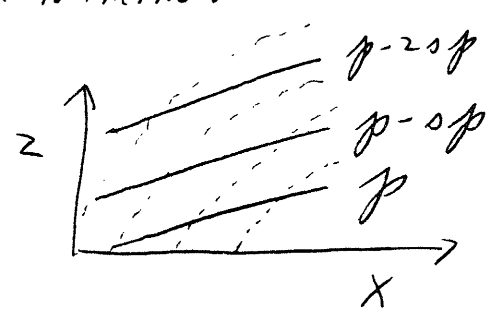
Obviously, a barotropic atmosphere rarely occurs. To some degree, the tropics is barotropic. However, much can be learned by studying a hypothetical barotropic atmosphere.

Some computer simulations only consider an <u>autobarotropic atmosphere</u>. This is an atmosphere which always remains barotropic. Such barotropic models cannot develop systems, but systems move and math is simple.

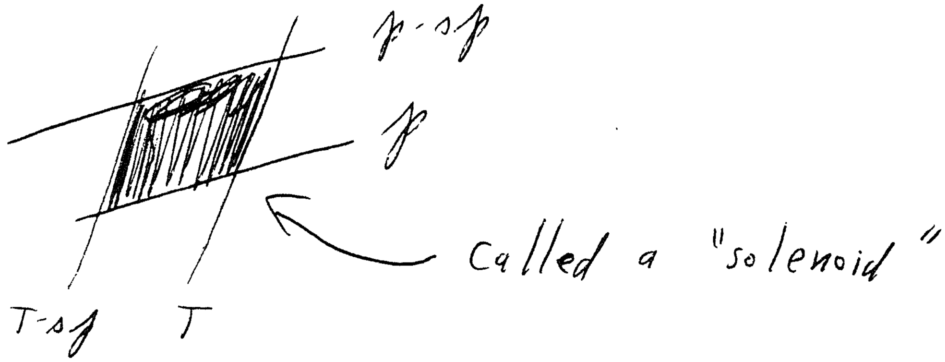
<u>Baroclinic atmosphere</u>

This is the real world, and is opposite of barotropic. Density depends on both temperature and pressure, $\rho = f(p, T)$. No variables coincide, and intersections occur between isobars and isotherms.





It is useful to "measure" how baroclinic the atmosphere is. One way is to count the number of intersections of p and T.



$$\text{baroclinicity} \equiv \frac{\text{number of solenoids}}{\text{unit area}}$$

It can be shown that:

1) The atmosphere is more baroclinic in winter than summer

2) A front is a highly baroclinic region. Hence, sometimes a frontal zone is also called a baroclinic zone

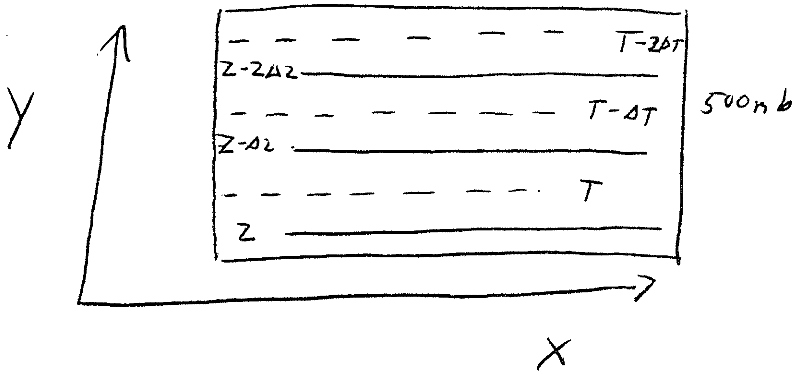
3) High baroclinicity is associated with areas of development.

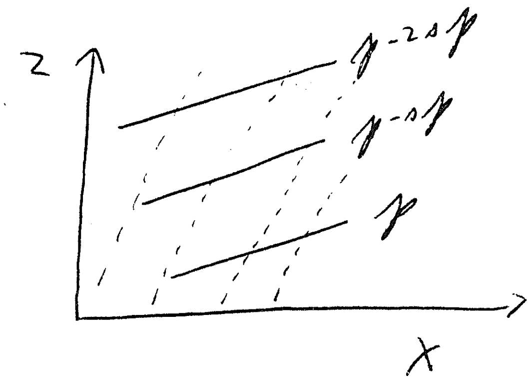
4) The thermal wind is large in highly baroclinic regions.

"Equivalent barotropic"

Sometimes in inactive weather regions, isotherms are parallel to geopotential heights. For example:

T and Z are "in phase"





Since no temperature advection is occurring, this situation is called "equivalent barotropic." However, the atmosphere is still baroclinic. In a truly barotropic atmosphere, T = constant at all pressure levels. $\vec{V}_g$ still changes ~~with height~~ speed with height. However, the direction remains constant with height. An "equivalent barotropic" atmosphere may occassionally be seen on maps, particularly at 500 mb.