Total derivative and advection

Some scalar field variable "A" at a location (x,y,z) which is a function of time t.

The change of A may be expressed as

$$\delta A = \left(\frac{\partial A}{\partial t}\right)\delta t + \left(\frac{\partial A}{\partial x}\right)\delta x + \left(\frac{\partial A}{\partial y}\right)\delta y + \left(\frac{\partial A}{\partial z}\right)\delta z$$

Divide by δt and take limit

$$\frac{DA}{Dt} \equiv \lim_{\delta t \to 0} \frac{\delta T}{\delta t}$$

So that

$$\frac{DA}{Dt} = \frac{\partial A}{\partial t} + \left(\frac{\partial A}{\partial x}\right)\underbrace{\frac{Dx}{Dt}}_{u} + \left(\frac{\partial A}{\partial y}\right)\underbrace{\frac{Dy}{Dt}}_{v} + \left(\frac{\partial A}{\partial z}\right)\underbrace{\frac{Dz}{Dt}}_{w}$$

rate a parcel crosses a height sfc

$$\underbrace{\frac{DA}{Dt}} = \frac{\partial A}{\partial t} + \underbrace{u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z}}_{\vec{V}\cdot\nabla A}$$

Total derivative
Material
Substantial.
Time Change following

local
rate
of change
of A.
Change of
A at a

advection
of A.
Represents how $A(x,y,z)$
will be redistributed by

Other tidbits

in x, y, p (on an isobaric surface)

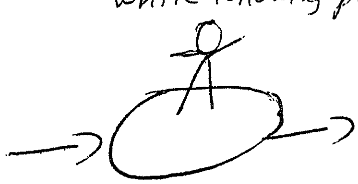
$$\frac{DA}{Dt} = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + \omega\frac{\partial A}{\partial p}$$

$\frac{Dp}{Dt}$) rate a parcel crosses a pressure surface.

$\frac{DA}{Dt} = 0$ means A is a "conserved" quantity. In other words, A = constant value following a fluid parcel.

$$\frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z} = 0$$

measure A while following parcel

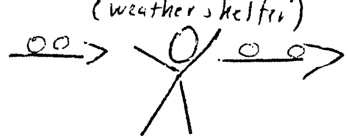


"Surfing" an air parcel.

Lagrangian

frame of reference

measure A at a fixed pt (weather shelter)



Eulerian frame of reference

$$\frac{\partial A}{\partial t}$$