Total derivative and advection

~~A~~

Some ~~A~~ scalar field variable "A" at a location (x, y, z) which is a function of time t.

The change of A may be expressed as

$$\delta A = \left(\frac{\partial A}{\partial t}\right)\delta t + \left(\frac{\partial A}{\partial x}\right)\delta x + \left(\frac{\partial A}{\partial y}\right)\delta y + \left(\frac{\partial A}{\partial z}\right)\delta z$$

Divide by δt and take limit

$$\frac{DA}{Dt} \equiv \lim_{\delta t \to 0} \frac{\delta T}{\delta t}$$

So that

$$\frac{DA}{Dt} = \frac{\partial A}{\partial t} + \left(\frac{\partial A}{\partial x}\right)\underbrace{\frac{Dx}{Dt}}_{u} + \left(\frac{\partial A}{\partial y}\right)\underbrace{\frac{Dy}{Dt}}_{v} + \left(\frac{\partial A}{\partial z}\right)\underbrace{\frac{Dz}{Dt}}_{w}$$

rate a parcel crosses a height sfc

$$\underbrace{\frac{DA}{Dt}}_{} = \frac{\partial A}{\partial t} + \underbrace{u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z}}_{\vec{V}\cdot\nabla A}$$

Total derivative
Material
Substantial.
Time change following

local
rate
of change
of A.
Change of
A at a

advection
of A.
Represents how $A(x,y,z)$
will be redistributed by

Other ~~comments~~: tidbits

in x, y, p (on an isobaric surface)

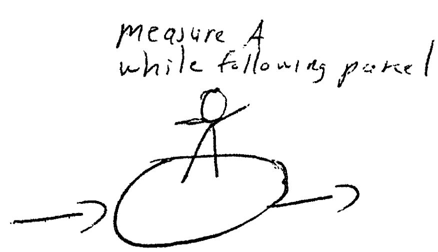
$$\frac{DA}{Dt} = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + \omega\frac{\partial A}{\partial p}$$

$\omega = \frac{Dp}{Dt}$) rate a parcel crosses a pressure surface.

$$\cancel{\frac{D(\;)}{Dt} =}$$

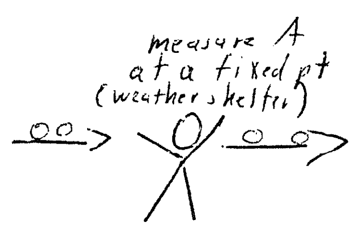
$\frac{DA}{Dt} = 0$ means A is a "conserved" quantity. In other words, A = constant value following a fluid parcel.

$$\frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z} = 0$$



"Surfing" an air parcel.

Lagrangian ~~...~~ frame of reference



Eulerian frame of reference

$$\frac{\partial A}{\partial t}$$

Example: temperature

Eq 2.42 in Holton

Thermodynamic energy equation:

(x, y, z) $$\frac{DT}{Dt} = \frac{\dot{Q}}{c_p} + \frac{1}{\rho c_p}\frac{Dp}{Dt}$$

$\underset{\text{me}}{\frac{\dot{Q}}{c_p}} = \underset{\text{Holton}}{J/c_p}$ = diabatic heating rate

a) Radiative heating (incoming SW radiation or greenhouse effect)
b) Radiative cooling (~~LW~~ outgoing LW radiation to space)
c) Phase change (latent heat of condensation or evaporation)
d) Heat fluxes or sinks (cold air moving over ocean, warming the air mass)
e) is zero under adiabatic conditions...

$\frac{1}{\rho}\frac{Dp}{Dt}$ = adiabatic temperature change due to expansion or compression

For example:

Under adiabatic conditions (overcast day in winter) with constant pressure, T is conserved!

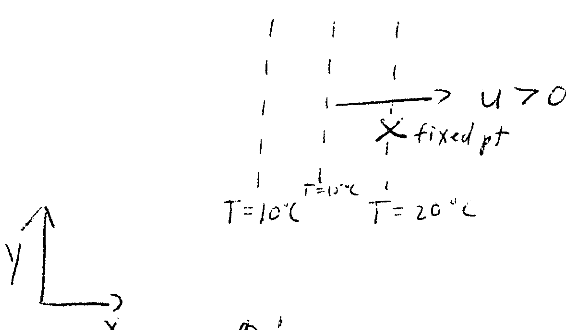
$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = 0$$

Therefore, the only local change in T $\left(\frac{\partial T}{\partial t}\right)$ at a fixed point is by temperature advection. Keep in mind that T following all parcels remains constant $\left(\frac{DT}{Dt}=0\right)$, but T is changing at fixed points since the wind is redistributing the temperature field with time.

Let's solve for $\frac{\partial T}{\partial t}$

$$\frac{\partial T}{\partial t} = \underbrace{-u\frac{\partial T}{\partial x}}_{\substack{\text{change in}\\ T \text{ by}\\ \text{zonal wind}\\ \text{advection}}} \underbrace{-v\frac{\partial T}{\partial y}}_{\substack{\text{change in}\\ T \text{ by}\\ \text{meridional}\\ \text{wind}\\ \text{advection}}} \underbrace{-w\frac{\partial T}{\partial z}}_{\substack{\text{change in}\\ T \text{ by}\\ \text{vertical}\\ \text{wind}\\ \text{advection}}}$$

Consider the hypothetical scenario where only a zonal wind exists and temperature decreases westward.



Obviouse, at pt X the 20°C isotherm will eventually be replaced by the 15°C isotherm, then the 10°C isotherm, as the temperature field gets displaced eastward by the west wind. This is <u>cold air advection</u>

Mathematically, this may be quantified as

This is the 1D temp advection equation →

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x}$$

The ~~following~~ variables exhibit the following signs:

$u > 0$

$\frac{\partial T}{\partial x} > 0$

$$\frac{\partial T}{\partial t} = -(+)(+) < 0$$

Since $-u \frac{\partial T}{\partial x} < 0$

Note the minus sign is included in the mathematical definition of advection.

In synoptic we'll see how to apply advection towards weather analysis