we discussed the adiabatic method for determining vertical velocities—a method based on the first law of thermodynamics; we also discussed the kinematic method—based on the equation of continuity. Now we shall develop a method for determining vertical motion based on the vorticity equation.

In isobaric coordinates we have

$$\frac{\partial}{\partial t}(\zeta_p + f) + \mathbf{v} \cdot \nabla_p(\zeta_p + f) + \omega \frac{\partial}{\partial p}(\zeta_p + f) = (\zeta_p + f)\frac{\partial \omega}{\partial p} + \frac{\partial \omega}{\partial y}\frac{\partial u}{\partial p} - \frac{\partial \omega}{\partial x}\frac{\partial v}{\partial p} \quad (1)$$

in which we have used the isobaric continuity equation to obtain the first term on the right.

The tilting terms in Eq. (1) are not necessarily small, as we demonstrated in Sec. 10.2, but to try to include them now would lead us far beyond the development of a simple technique. Thus they are dropped at this point solely for computational convenience. Of course, if they are large in a given case, then the vertical velocities produced by this method will be in error. With the further assumption that $\zeta_p + f > 0$ we may multiply Eq. (1) by $(\zeta + f)^{-2}$ and thus obtain

$$\frac{\partial}{\partial p}\left(\frac{\omega}{\zeta_p + f}\right) = \frac{1}{(\zeta_p + f)^2}\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_p\right)(\zeta_p + f) \quad (2)$$

Upon integration from level p_0 down to level p we have

$$\omega(p) = (\zeta_p + f)\left[\frac{\omega_0}{(\zeta_p + f)_{p_0}} + \int_{p_0}^{p} \frac{1}{(\zeta_p + f)^2}\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{p'}\right)(\zeta_{p'} + f)\, dp'\right] \quad (3)$$

in which p' is the variable of integration. If we choose $p_0 = 0$, then ω_0 vanishes and so

$$\omega(p) = (\zeta_p + f)\int_0^p \frac{1}{(\zeta_{p'} + f)^2}\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{p'}\right)(\zeta_{p'} + f)\, dp' \quad (4)$$

If we therefore evaluate the local change of vorticity from successive observations and also compute the vorticity advection on many levels from p to the upper part of the atmosphere, we can determine the fields of isobaric vertical velocity on level p. The absence of the tilting terms is a source of error, and so is the fact that the local change and advection of vorticity tend to cancel—as might be expected from the discussion in Secs. 10.3 and 10.4.

The result (4) provides a basis for a concept employed by forecasters. If strong, positive advection of vorticity is observed in the upper part of the atmosphere, say above 500 mbar, then there will be a strong contribution to negative ω, that is, upward vertical motion at lower levels. Thus the identification of regions of positive vorticity advection at upper levels is a valuable guide to prediction of precipitation. As can be shown from Eq. (3), negative vorticity advection at low levels (with $\omega_0 \sim 0$) gives the same result. Hence, forecasters watch for situations in which the advection of vorticity changes sign between about 700 and 500 mbar. Observe, though, that the local derivative and the advection in Eq. (4) tend to cancel each other, so that the argument above must be refined by considering the effect of

1) temperature advection
2) diabatic heating
3) boundary layer friction