Thermodynamic energy equations

\[ \frac{\partial T}{\partial t} = \frac{Q}{c_p} + \frac{1}{\rho c_p} \frac{\partial p}{\partial t} \]

\[ Q = \dot{Q}_{\text{le}} \] diabatic heating rate

\[ \text{due to radiation} \]

a) Radiative heating (SW or greenhouse effect)
b) Radiative cooling (ciling LW radiation to space)
c) Phase change (latent heat of condensation or evaporation)
d) Heat fluxes (cold air moving over warm, warming the air mass)
e) is zero under adiabatic conditions...

\[ \frac{1}{\rho} \frac{\partial p}{\partial t} = \text{adiabatic temperature change due to expansion or compression} \]

For example:

Under adiabatic conditions (overcast day in winter) with constant pressure, \( T \) is conserved!

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \nabla \cdot \nabla T = 0 \]
Therefore, the only local change in $T$ ($\frac{\partial T}{\partial t}$) at a fixed point is by temperature advection. Keep in mind that $\frac{\partial T}{\partial t} = 0$, but $T$ is changing at fixed points since the wind is redistributing the temperature field with time.

Let's solve for $\frac{\partial T}{\partial t}$:

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} - w \frac{\partial T}{\partial z},$$

- change in $T$ by zonal wind advection
- change in $T$ by meridional wind advection
- change in $T$ by vertical wind advection

Consider the hypothetical scenario where only a zonal wind exists and temperature decreases westward:

$u > 0$ fixed pt

$T = 10^\circ C$ $\rightarrow$ $T = 20^\circ C$

Obviously, at pt $x$ the $20^\circ C$ isotherm will eventually be replaced by the $15^\circ C$ isotherm, then the $10^\circ C$ isotherm, as the temperature field gets displaced eastward by the west wind. This is cold air advection.
Mathematically, this may be quantified as

\[
\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x}
\]

The variables exhibit the following signs:

\[u > 0\]
\[\frac{\partial T}{\partial x} > 0\]

\[\frac{\partial T}{\partial t} = -(+) (+) < 0\]

Since \[ -u \frac{\partial T}{\partial x} < 0\]

Note the minus sign is included in the mathematical definition of advection.

In synoptic we'll see how to apply advection towards weather analysis.