Finite difference approximation

Suppose $f(x) = x^3$

The derivative is $f'(x) = 3x^2$

At $x = 1.5$, $f'(x) = 3(1.5)^2 = 6.75$

If we compute $f(x)$ to the right and left of $x = 1.5$, equally spaced, we can approximate the slope. Examples are shown below, with progressively smaller spacing around $x = 1.5$.

$$f'(x=1.5) \approx \frac{f(2.0) - f(1.0)}{2.0 - 1.0} = 7.0 \text{, since } \frac{(2)^3 - (1)^3}{2-1} = \frac{8-1}{1}$$

Likewise

$$f'(x=1.5) \approx \frac{f(1.75) - f(1.25)}{1.75 - 1.25} = 6.8 \text{, since } \frac{(1.75)^3 - (1.25)^3}{1.75 - 1.25}$$

$$= \frac{5.36 - 1.95}{0.5}$$

$$f'(x=1.5) \approx \frac{f(1.6) - f(1.4)}{1.6 - 1.4} = 6.76 \text{, since } \frac{(1.6)^3 - (1.4)^3}{1.6 - 1.4}$$

$$= \frac{4.096 - 2.744}{0.2}$$

Formally, a centered finite difference can be written as

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

As Δx gets smaller, the approximation for $f'(x)$ gets more accurate. This is consistent with the formal definition of the derivative.