

Compute the derivative of $f(x) = 6x^2 + 10x + 3$

1) Using the formal definition in which $\lim_{\Delta x \rightarrow 0}$

The formal definition is

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$$

$$\lim_{\Delta x \rightarrow 0} \frac{[6(x+\Delta x)^2 + 10(x+\Delta x) + 3] - [6x^2 + 10x + 3]}{\Delta x} =$$

$$\lim_{\Delta x \rightarrow 0} \left\{ \frac{6(x^2 + 2x\Delta x + (\Delta x)^2) + 10(x + \Delta x) + 3 - 6x^2 - 10x - 3}{\Delta x} \right\} =$$

Collect terms, then cancel where applicable

$$\lim_{\Delta x \rightarrow 0} \left\{ \frac{\cancel{6x^2} + 12x\Delta x + 6(\Delta x)^2 + \cancel{10x} + 10\Delta x + \cancel{3} - \cancel{6x^2} - \cancel{10x} - \cancel{3}}{\Delta x} \right\} =$$

Divide by Δx

$$\lim_{\Delta x \rightarrow 0} \{ 12x + 6\Delta x + 10 \}$$

Apply $\lim_{\Delta x \rightarrow 0}$

$$\lim_{\Delta x \rightarrow 0} \{ 12x + 6\Delta x + 10 \}$$

↓
0

Which gives the final result

$$f'(x) = \frac{df(x)}{dx} = 12x + 10$$

Compute the derivative of $f(x) = 6x^2 + 10x + 3$
using the shortcut

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

$$f'(x) = \frac{df(x)}{dx} = (2)6x^{2-1} + (1)10x^{1-1}$$

↖ $x^0 = 1$

$$f'(x) = \frac{df(x)}{dx} = 12x + 10$$

Compute $f'(x)$ at $x = -0.3$

$$f'(-0.3) = 12(-0.3) + 10 = \boxed{6.4}$$