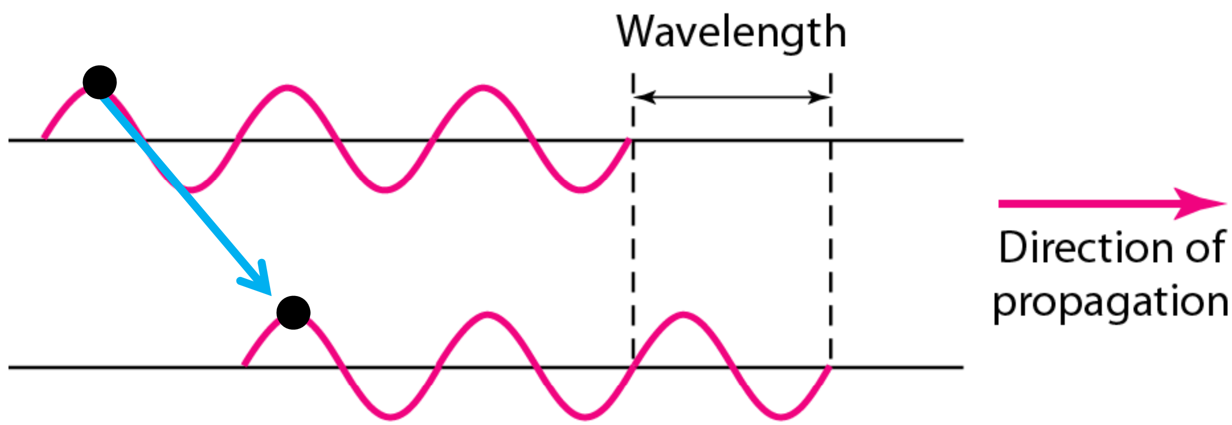
Phase Speed

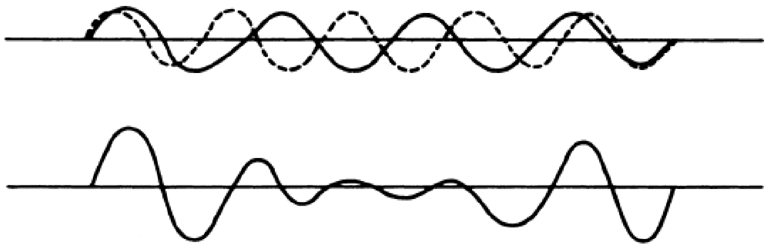
- The phase velocity of a wave is the rate at which the phase of the wave propagates in space.
- The phase speed is given in terms of the wavelength λ and period T (or frequency ν and wavenumber k) as:

$$c = \frac{\lambda}{T}.$$

$$c = \nu / k$$



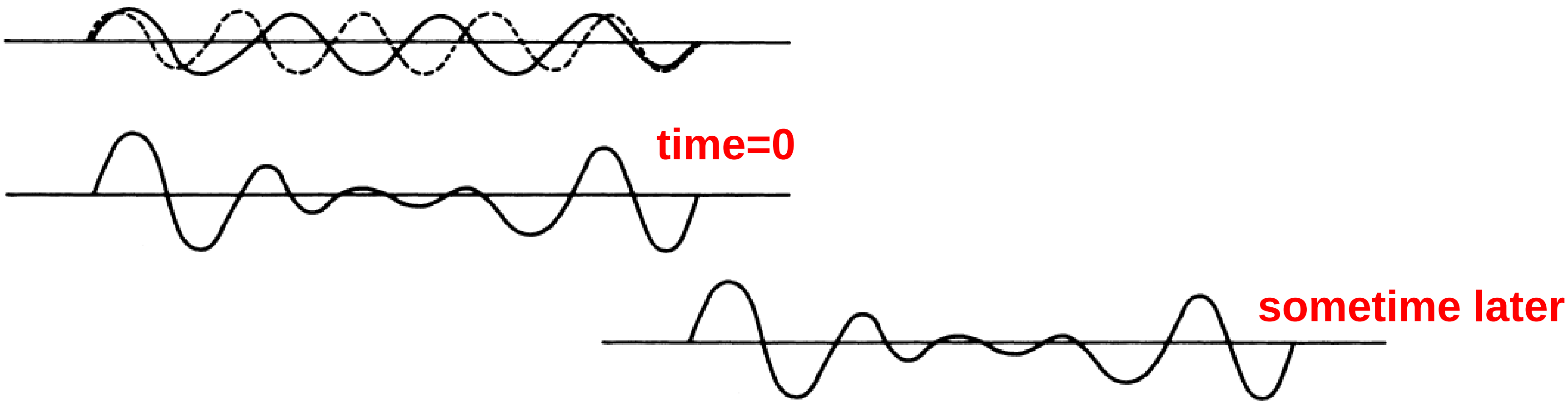
A Group of Waves with Different Wavenumbers



$$c = \nu / k$$

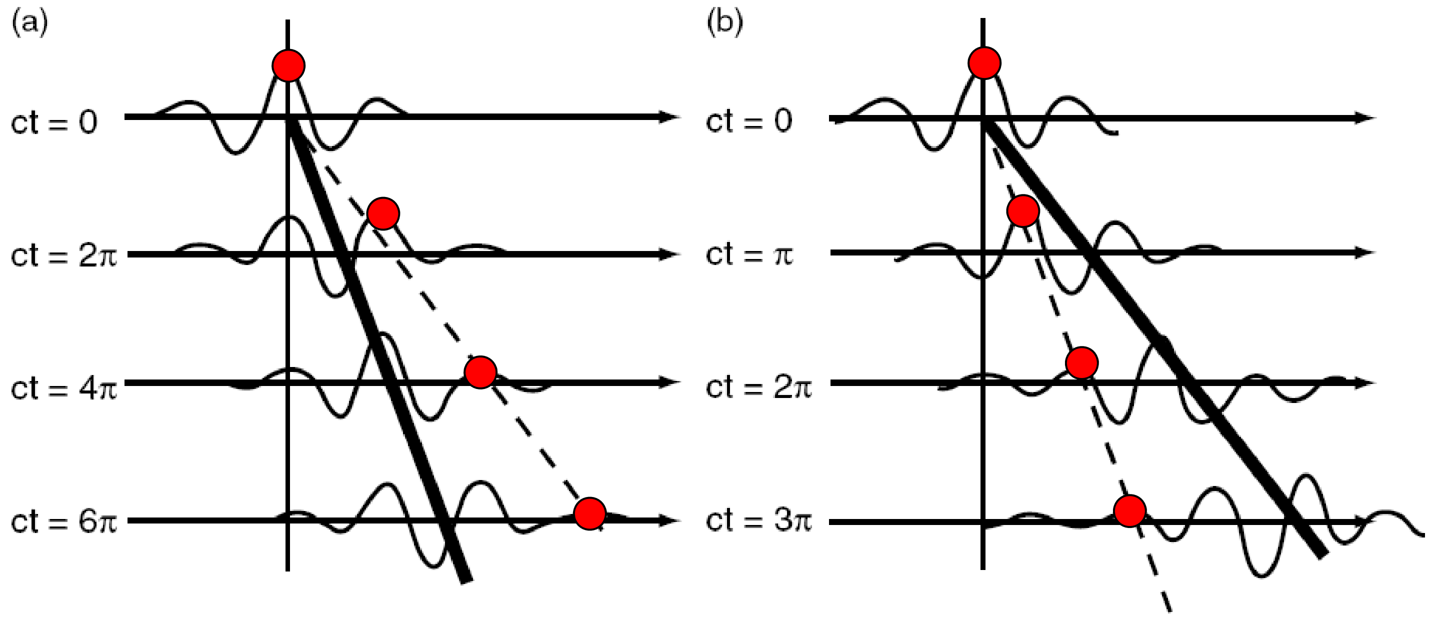
- In cases where several waves add together to form a single wave shape (called the **envelope**), each individual wave component has its own wavenumber and phase speed.
- For waves in which the phase speed varies with k, the various sinusoidal components of a disturbance originating at a given location are at a later time found in different places. Such waves are *dispersive*.
- For *nondispersive* waves, their phase speeds that are independent of the wave number.

Non-Dispersive Waves



- Some types of waves, such as acoustic waves, have phase speeds that are independent of the wave number.
- In such *nondispersive waves a spatially* localized disturbance consisting of a number of Fourier wave components (a *wave group) will preserve its shape as it propagates in space at the phase speed of the* wave.

Dispersive Waves



- For dispersive waves, the shape of a wave group will not remain constant as the group propagates.
- Furthermore, the group generally broadens in the course of time, that is, the energy is *dispersed.*
- When waves are dispersive, the speed of the wave group is generally different from the average phase speed of the individual Fourier components.
- In synoptic-scale atmospheric disturbances, however, the group velocity exceeds the phase velocity.

Group Velocity

$$c_{gx} = \partial \nu / \partial k$$

- The group velocity of a wave is the velocity with which the overall shape of the wave's amplitudes (i.e. envelope) propagates through space.
- Two horizontally propagating waves of equal amplitude but slightly different wavelengths with wave numbers and frequencies differing by $2\delta k$ and $2\delta v$, respectively. The total disturbance is thus:

$$\Psi(x,t) = \exp\{i[(k+\delta k)x - (\nu+\delta\nu)t]\} + \exp\{i[(k-\delta k)x - (\nu-\delta\nu)t]\}$$

$$= \left[e^{i(\delta k x - \delta\nu t)} + e^{-i(\delta k x - \delta\nu t)}\right] e^{i(kx-\nu t)}$$

$$= 2\cos(\delta k x - \delta\nu t)\, e^{i(kx-\nu t)}$$

low-frequency amplitude modulation

high-frequency carrier wave

travels at the speed of *$\delta v/\delta k$* ➔ *group velocity*