7.1.2 Tangential Cartesian Coordinates

The most common meteorological coordinate system and equations are obtained from the previous results by placing an orthogonal cartesian system with its origin at a point on the earth's surface, say $\mathbf{x}_0$. We take the z axis parallel to the radial vector with z increasing away from the earth's surface. The x and y axes form a plane tangent to the earth at $\mathbf{x}_0$, and we let x increase toward the east and let y increase toward the north. The advantage of this system is that it is an orthogonal cartesian system that includes the earth's rotation. Furthermore, it is a system that corresponds to our own natural references, and it is the one that is necessarily appropriate to observations made with radio and radar techniques.

In order to apply the equation of motion (18) in this system we shall need the components of the Coriolis acceleration. From Fig. 7.5 it is easily seen that

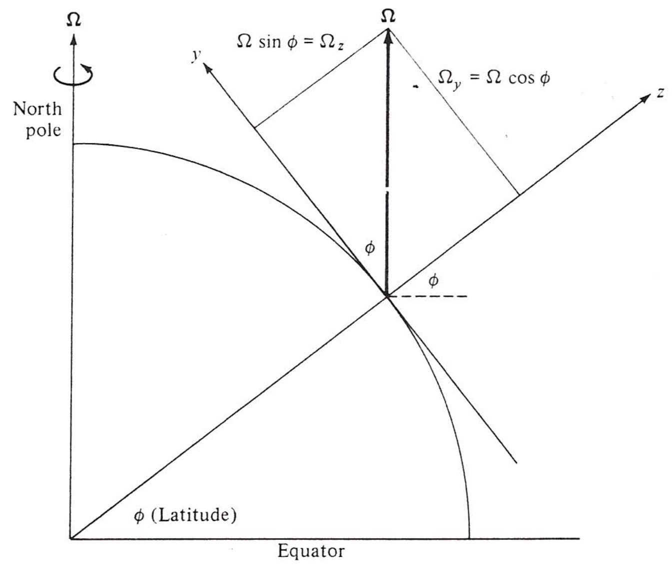

FIGURE 7.5
Projection of the vector $\mathbf{\Omega}$ on the y and z axes to determine its components.

$$\Omega_x = 0 \qquad \Omega_y = \Omega \cos \phi \qquad \Omega_z = \Omega \sin \phi \tag{19}$$

in which ϕ is the latitude.† Thus we have

$$-2\vec{\mathbf{\Omega}} \times \vec{\mathbf{v}} = -2\Omega \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \cos\phi & \sin\phi \\ u & v & w \end{vmatrix} \tag{20}$$

The scalar equation for any component can be obtained by taking the scalar product of Eqs. (18) and (20) with the appropriate unit vector.

To apply these results we must know the magnitude of Ω as measured with respect to the fixed stars. In Chap. 2 we pointed out that there is one more sidereal day than solar day in each year owing to the earth's revolution around the sun (see Fig. 7.6). Thus we have

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$$\begin{aligned} \Omega &= \frac{2\pi}{1 \text{ sidereal day}} \cdot \frac{366.25 \text{ sidereal days}}{365.25 \text{ solar days}} = 2\pi \frac{366.25}{365.25} (\text{solar days})^{-1} \\ &= \frac{2\pi}{86{,}400 \text{ s}} \cdot \frac{366.25}{365.25} = 7.292 \times 10^{-5}/\text{s} \end{aligned} \tag{21}$$

†Note that if we set $\phi = \pi/2$ at the North Pole, $\phi = 0$ at the equator, and $\phi = -\pi/2$ at the South Pole, then the Coriolis force will have the correct sign in both Northern and Southern Hemispheres.

Taken from "The Ceaseless Wind" by John Dutton

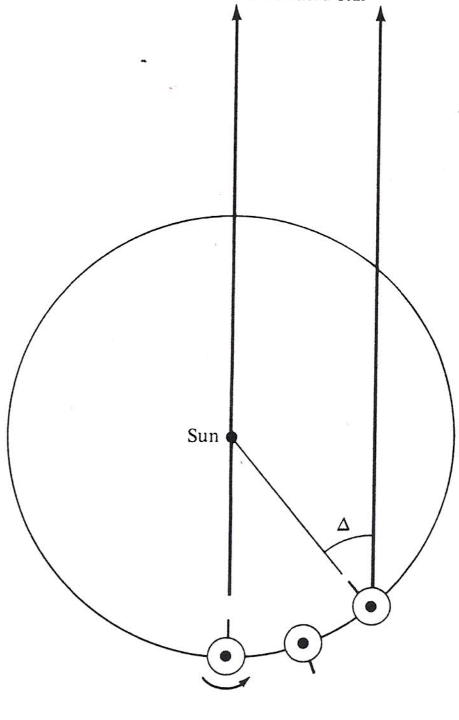

FIGURE 7.6
Schematic comparison of solar and sidereal days. In the interval shown, the earth has made one complete rotation with respect to the sun; hence one solar day has elapsed. But in the process it has rotated once plus the angle Δ with respect to the fixed star. Thus more than one sidereal day has elapsed. As can be seen, Δ will be 2π after one revolution around the sun, giving one more sidereal day in a year than there are solar days.

Therefore we obtain

$$\begin{aligned} 2\Omega_y &= 2\Omega \cos\phi = 1.458 \times 10^{-4} \cos\phi/\mathrm{s} \\ 2\Omega_z &= 2\Omega \sin\phi = 1.458 \times 10^{-4} \sin\phi/\mathrm{s} \end{aligned} \qquad (22)$$

and so at $\phi = 45°$, we have $2\Omega_y = 2\Omega_z \cong 10^{-4}/\mathrm{s}$.

This coordinate system with ϕ set constant at ϕ_0 is useful for small-scale local studies and provides a simple system in which to perform theoretical analyses of the equations of motion with the effects of rotation included; it is not well suited for practical applications to large-scale problems for two reasons associated with the fact that the earth is spherical. The first is that a purely zonal flow will have only an eastward component at $\mathbf{x}_0$, but as the air proceeds east, it will fall below the tangent plane and acquire a z component of motion in this system. The second is that for large-scale problems we shall have to include the latitudinal variation of the Coriolis forces.

Note: The solar day is the interval required for one earth rotation with respect to the sun; the solar year is the interval required for one revolution around the sun. The sidereal day is the interval required for one rotation with respect to a distant star. Because of the earth's revolution, there is one more sidereal day than there are solar days per year.

From Chapter 2

Solar day = 24 h
Sidereal day = 23 h 56 min