Gradient Wind

The geostrophic balance is defined as motion parallel to isobars in which there is a balance between the pressure gradient force and Coriolis force. Frictionless, straight-line flow is assumed.

However, when the flow is curved, the centrifugal force must also be considered. Wind blowing parallel to isobars in curved flow, such as a trough, cyclone, ridge, or anticyclone is called the <u>gradient wind</u>, (where $\frac{d|\vec{V}|}{dt} = 0 = \frac{-\partial \Phi}{\partial s}$. For gradient wind, there is a three-way balance between the Coriolis force, the pressure gradient force, and the centrifugal force.

Recall the following relationship:

$$(1) \quad \underbrace{\frac{|\vec{V}|^2}{R}}_{CeF} + \underbrace{f|\vec{V}|}_{CoF} = \underbrace{-\frac{\partial \Phi}{\partial n}}_{PGF}$$

Which may be rewritten as

$$(2) \quad |\vec{V}|^2 + fR|\vec{V}| + R\frac{\partial \Phi}{\partial n} = 0$$

by multiplying (1) by R

2 may be solved for $|\vec{V}|$ using the quadratic eq:

$$(3) \quad |\vec{V}| = \overbrace{\frac{-fR}{2}}^{\text{term } i} \pm \overbrace{\sqrt{\underbrace{\left(\frac{fR}{2}\right)^2}_{A} - \underbrace{\left(R\frac{\partial \Phi}{\partial n}\right)}_{B}}}^{\text{term } ii}$$

where the quantity under the root must be positive. In other words, term A − term B > 0
Also, $|\vec{V}| > 0$ (term i ± term ii > 0). Hence $|\vec{V}|$ must be real, and positive definite.

However, 3 is more complicated than it seems. This is because the following conditions must be considered:

1) Cyclonic flow ($R > 0$)
2) Anticyclonic flow ($R < 0$)
3) $\frac{\partial \Phi}{\partial n} > 0$
4) $\frac{\partial \Phi}{\partial n} < 0$
5) The positive sign in front of the square root,
6) The negative sign in front of the square root.

In other words, there are four cyclonic possibilities and four anticyclonic possibilities:

Cyclonic, $R > 0$	Anticyclonic, $R < 0$
$\frac{\partial \Phi}{\partial n} > 0$, $+\sqrt{\quad}$	$\frac{\partial \Phi}{\partial n} > 0$, $+\sqrt{\quad}$
$\frac{\partial \Phi}{\partial n} < 0$, $+\sqrt{\quad}$	$\frac{\partial \Phi}{\partial n} < 0$, $+\sqrt{\quad}$
$\frac{\partial \Phi}{\partial n} > 0$, $-\sqrt{\quad}$	$\frac{\partial \Phi}{\partial n} > 0$, $-\sqrt{\quad}$
$\frac{\partial \Phi}{\partial n} < 0$, $-\sqrt{\quad}$	$\frac{\partial \Phi}{\partial n} < 0$, $-\sqrt{\quad}$

Consider the cyclonic cases (with $R > 0$)

I $\frac{\partial \Phi}{\partial n} > 0$, $+\sqrt{}$

Since $|\text{term } i| > |\text{term } ii|$

$$|\vec{V}| = -\frac{fR}{2} + \sqrt{\left(\frac{fR}{2}\right)^2 - \left(R\frac{\partial \Phi}{\partial n}\right)} < 0 \quad \text{always!}$$

This root is <u>unphysical</u>

II $\frac{\partial \Phi}{\partial n} > 0$, $-\sqrt{}$

Since term $i < 0$, term $ii < 0$

$$|\vec{V}| = -\frac{fR}{2} - \sqrt{\left(\frac{fR}{2}\right)^2 - \left(R\frac{\partial \Phi}{\partial n}\right)} < 0 \quad \text{always!}$$

This root is <u>unphysical</u>

Cases I and II are cyclonic rotation about a high.

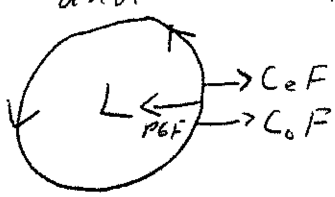
Note that in cases I and II, the PGF, CeF, and C.F are all acting outwards. It is impossible to have a balance of forces in this case, hence both solutions aren't possible!

III $\frac{\partial \Phi}{\partial n} < 0$, $+\sqrt{\quad}$

Since $-R\frac{\partial \Phi}{\partial n} > 0$ always (term B), the value under the $\sqrt{\quad}$ is always > 0. Furthermore $|\text{term ii}| > |\text{term i}|$ always. Hence

$$|\vec{V}| = -\frac{fR}{2} + \sqrt{\left(\frac{fR}{2}\right)^2 - R\frac{\partial \Phi}{\partial n}} > 0 \quad \text{always!}$$

This is a regular low, since Φ decreases toward the center and flow is cyclonic



Note: CeF always acts outwards from curved flow

IV $\frac{\partial \Phi}{\partial n} < 0$, $-\sqrt{\quad}$

Since term i < 0, term ii < 0

$$|\vec{V}| = -\frac{fR}{2} - \sqrt{\left(\frac{fR}{2}\right)^2 - \left(R\frac{\partial \Phi}{\partial n}\right)} < 0 \quad \text{always}$$

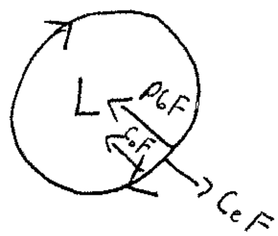
This root is unphysical

Now consider the anticyclonic cases (with $R<0$)

I) $\frac{\partial \Phi}{\partial n} > 0$, $+\sqrt{\quad}$

Since $-R\frac{\partial \Phi}{\partial n} > 0$ always (term B), the value under $\sqrt{\quad}$ is always >0. Furthermore, term i >0. Hence $|\vec{V}| = -\frac{fR}{2} + \sqrt{\left(\frac{fR}{2}\right)^2 - R\frac{\partial \Phi}{\partial n}} > 0$ always!

This is an actual solution, which states it is possible for anticyclonic flow to exist around a low pressure system.



Because $\hat{n}$ always points to the left of flow, Φ actually decreases toward the center.

This is an anomalous low, since it theoretically could exist but is not observed.

Note also that for straight flow, $|\vec{V}_g| < 0$, which is another sign this is not reality!

II) $\frac{\partial \Phi}{\partial n} > 0$, $-\sqrt{\quad}$

Since $-R\frac{\partial \Phi}{\partial n} > 0$, |term ii| > |term i|. Therefore $|\vec{V}| = -\frac{fR}{2} - \sqrt{\left(\frac{fR}{2}\right)^2 - R\frac{\partial \Phi}{\partial n}} < 0$ always! (since term i >0)

This root is unphysical!

III $\frac{\partial \Phi}{\partial n} < 0$, $+\sqrt{\quad}$

Since $-R\frac{\partial \Phi}{\partial n} < 0$, $|\text{term i}| > |\text{term ii}|$.

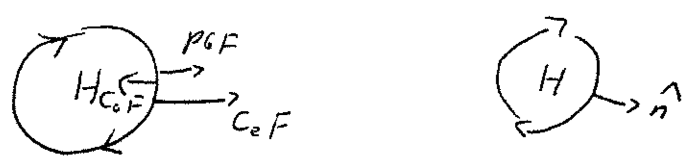
Furthermore, $-\frac{fR}{2} > 0$, so term i > 0. Hence

$|\vec{V}| = -\frac{fR}{2} + \sqrt{\left(\frac{fR}{2}\right)^2 - R\frac{\partial \Phi}{\partial n}} > 0$ always! (if $\left(\frac{fR}{2}\right)^2 > -R\frac{\partial \Phi}{\partial n}$)

This is an actual solution, which states it is possible to have fast anticyclonic flow! $|\vec{V}|$ is large because $-\frac{fR}{2} > 0$ and $+\sqrt{\quad}$

If one plugged in reasonable values for f, R, and $\frac{\partial \Phi}{\partial n}$, using $+\sqrt{\quad}$ would give anticyclonic wind values which are too fast compared to observations

This is an anomalous high. Fast anticyclonic flow is theoretically possible, but is not observed. Φ increases toward the center and flow is anticyclonic



Since the wind speed is fast, qualitatively CeF should be stronger than PGF or CoF.

Note that for $\frac{\partial \Phi}{\partial n} = 0$, $|\vec{V}| = -fR$, the velocity of inertial flow! Inertial flow is part of the solution of an anomalous high.

IV $\frac{\partial \Phi}{\partial n} < 0$, $-\sqrt{\quad}$

Since $-R\frac{\partial \Phi}{\partial n} < 0$, $|\text{term i}| > |\text{term ii}|$. Furthermore, $-\frac{fR}{2} > 0$, so term i > 0. Hence

$$|\vec{V}| = -\frac{fR}{2} - \sqrt{\left(\frac{fR}{2}\right)^2 - R\frac{\partial \Phi}{\partial n}} > 0 \text{ always! } \left(\text{If } \left(\frac{fR}{2}\right)^2 > -R\frac{\partial \Phi}{\partial n}\right)$$

This is a regular high, since Φ increases toward the center and flow is anticyclonic. However, since term ii < 0, this yields a weak anticyclonic flow. Weak flow around highs is observed in nature, so this is the true solution for an anticyclone.