Inertia - buoyancy waves

These waves horizontal scale >> vertical scale, so hydrostatic approx is valid. Eqs are $\bar{u}=0$

$$\frac{\partial u'}{\partial t} - fv' + \frac{1}{\rho}\frac{\partial p'}{\partial x} = 0$$

$$\frac{\partial v'}{\partial t} + fu + \frac{1}{\rho}\frac{\partial p'}{\partial y} = 0$$

$$\frac{1}{\rho}\frac{\partial p'}{\partial z} - \frac{\theta'}{\bar{\theta}} g = 0$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial}{\partial t}\left(\frac{1}{\rho}\frac{\partial p'}{\partial z}\right) + N^2 w' = 0$$

$$\frac{\partial \theta'}{\partial t} + w'\frac{\partial \bar{\theta}}{\partial z} = 0$$

Assume solutions $(u', v', w', p') = \hat{U}, \hat{V}, \hat{W}, \hat{P} \exp[i(kx+ly+mz-\omega t)]$

Get

$$\boxed{\nu^2 = N^2\frac{k^2+l^2}{m^2} + f^2}$$

which is similar to buoyancy wave

$$\nu^2 = \frac{N^2k^2}{m^2+k^2}$$

$\to m^2$ since $m^2 \gg k^2$ here.

$$\nu = \pm\sqrt{f^2 + N^2\frac{k^2+l^2}{m^2}}$$

For vertical propagation, $|f| < |\omega| << N$

$$C_{gx} = \frac{\partial \omega}{\partial k} = \frac{\pm\, kN^2/m^2}{\sqrt{f^2 + N^2\frac{k^2+l^2}{m^2}}}$$

$$C_{gz} = \frac{\partial \omega}{\partial m} = \frac{\pm -N^2 k^2 - l^2}{m^3\sqrt{f^2 + N^2\frac{k^2+l^2}{m^2}}}$$