Show that

$$\text{I} \quad \boxed{c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = 0}$$

Can be written as

$$\boxed{c_v \frac{D\ln p}{Dt} - c_p \frac{D\ln \rho}{Dt} = 0}$$

The trick is to rewrite $p = \rho R T$ using the $\ln$ identity

$$\ln(abc) = \ln(a) + \ln(b) + \ln(c)$$

Hence

$$\ln p = \ln \rho + \ln R + \ln T$$

Then take a total derivative

$$\boxed{\frac{D\ln p}{Dt} = \frac{D\ln \rho}{Dt} + \cancelto{0}{\frac{D\ln R}{Dt}} + \frac{D\ln T}{Dt}}$$

or solve for $\frac{D\ln T}{Dt}$

$$\text{II} \quad \boxed{\frac{D\ln T}{Dt} = \frac{D\ln p}{Dt} - \frac{D\ln \rho}{Dt}}$$

Note that I can be rewritten by substituting the ideal gas law as

$$\frac{1}{\rho} = \frac{RT}{p}$$

So that I can be written as

$$C_p \frac{DT}{Dt} - \frac{RT}{p}\frac{Dp}{Dt} = 0$$

or, divide by T

$$\frac{C_p}{T}\frac{DT}{Dt} - \frac{R}{p}\frac{Dp}{Dt} = 0$$

Note that $\frac{1}{a}\frac{Da}{Dt}$ can be written $\frac{D\ln(a)}{Dt}$

III $$\boxed{C_p \frac{D\ln T}{Dt} - R\frac{D\ln p}{Dt} = 0}$$

Substitute II into III

$$C_p \frac{D\ln p}{Dt} - C_p \frac{D\ln \rho}{Dt} - R\frac{D\ln p}{Dt} = 0$$

But $C_v = C_p - R$

$$\boxed{C_v \frac{D \ln p}{Dt} - C_p \frac{D \ln \rho}{Dt} = 0}$$

Proof completed