Derivation of the hypsometric equation:
Given: the hydrostatic eq:

$$\frac{dP}{dz} = -\rho \cdot |g| \qquad (1.25c)$$

and the ideal gas law:

$$P = \rho \cdot \Re_d \cdot T_v \qquad (1.23)$$

First, rearrange eq. (1.23) to solve for density:

$$\rho = P / (\Re_d \cdot T_v)$$

Then substitute this into (1.25c):

$$\frac{dP}{dz} = -\frac{P \cdot |g|}{\Re_d \cdot T_v}$$

One trick for integrating equations is to separate variables. Move all the pressure factors to one side, and all height factors to the other. Therefore, multiply both sides of the above equation by dz, and divide both sides by P.

$$\frac{dP}{P} = -\frac{|g|}{\Re_d \cdot T_v} dz$$

Compared to the other variables, g and $\Re_d$ are relatively constant, so we will assume that they are constant and separate them from the other variables. However, usually temperature varies with height: $T(z)$. Thus:

$$\frac{dP}{P} = -\frac{|g|}{\Re_d} \cdot \frac{dz}{T_v(z)}$$

Next, integrate the whole eq. from some lower altitude z_1 where the pressure is P_1, to some higher altitude z_2 where the pressure is P_2:

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\frac{|g|}{\Re_d} \cdot \int_{z_1}^{z_2} \frac{dz}{T_v(z)}$$

where $|g|/\Re_d$ is pulled out of the integral on the RHS because it is constant.

The left side of that equation integrates to become a natural logarithm (consult tables of integrals).

The right side of that equation is more difficult, because we don't know the functional form of the vertical temperature profile. On any given day, the profile has a complex shape that is not conveniently described by an equation that can be integrated.

Instead, we will invoke the mean-value theorem of calculus to bring T_v out of the integral. The overbar denotes an average (over height, in this context).

That leaves only dz on the right side. After integrating, we get:

$$\ln(P)\Big|_{P_1}^{P_2} = -\frac{|g|}{\Re_d} \cdot \overline{\left(\frac{1}{T_v}\right)} \cdot z\Big|_{z_1}^{z_2}$$

Plugging in the upper and lower limits gives:

$$\ln(P_2) - \ln(P_1) = -\frac{|g|}{\Re_d} \cdot \overline{\left(\frac{1}{T_v}\right)} \cdot (z_2 - z_1)$$

But the difference between two logarithms can be written as the ln of the ratio of their arguments:

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{|g|}{\Re_d} \cdot \overline{\left(\frac{1}{T_v}\right)} \cdot (z_2 - z_1)$$

Recalling that $\ln(x) = -\ln(1/x)$, then:

$$\ln\left(\frac{P_1}{P_2}\right) = \frac{|g|}{\Re_d} \cdot \overline{\left(\frac{1}{T_v}\right)} \cdot (z_2 - z_1)$$

Rearranging and approximating $\overline{1/T_v} \approx 1/\overline{T_v}$ (which is NOT an identity), then one finally gets the hypsometric eq:

$$(z_2 - z_1) \approx \frac{\Re_d}{|g|} \cdot \overline{T_v} \cdot \ln\left(\frac{P_1}{P_2}\right) \qquad (1.26)$$